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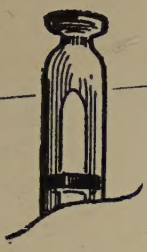

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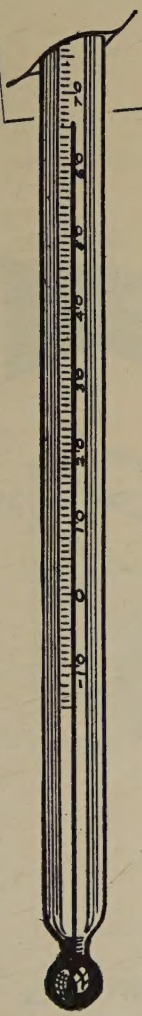

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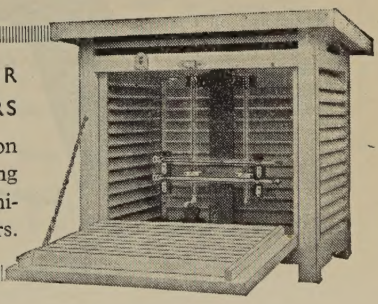

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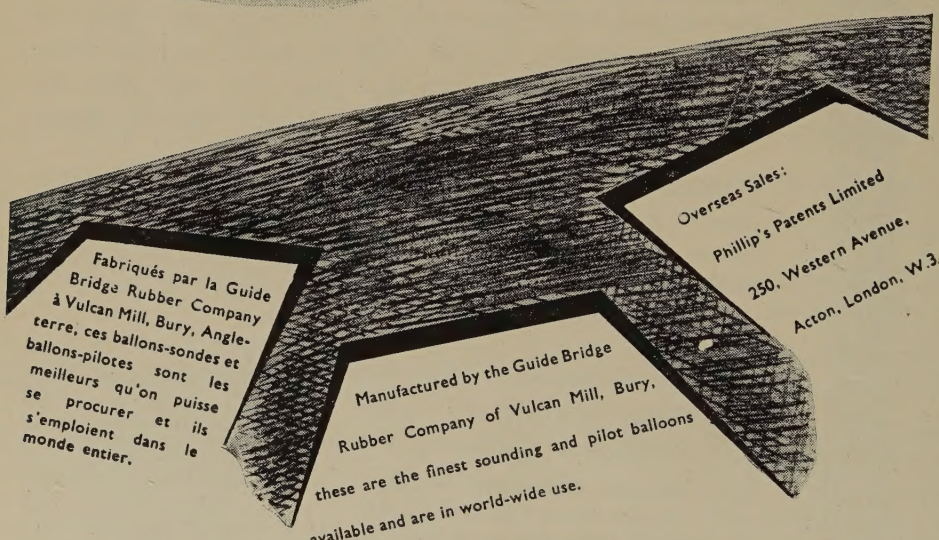

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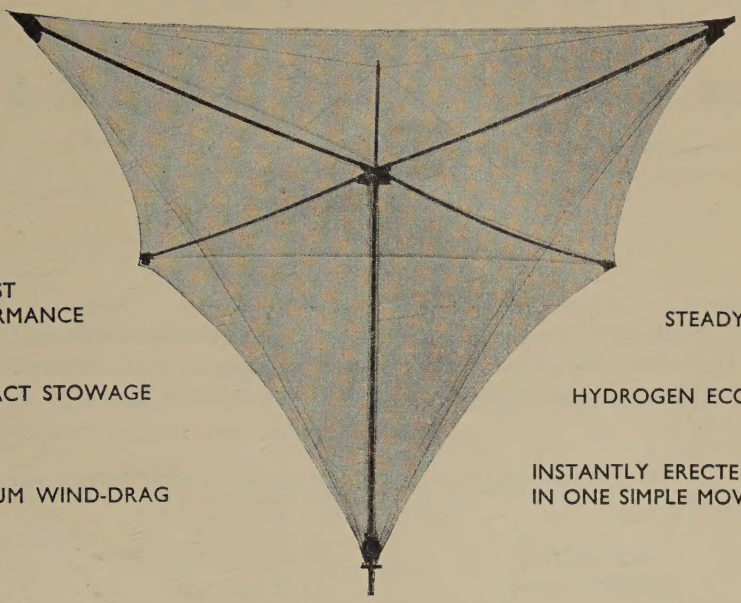

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Free and forced convection in the atmosphere near the ground

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(Manuscript received 20 September 1954)

SUMMARY

There is evidence for quite a sharp transition between the regimes of fully forced and of dominantly free convection in the atmosphere near the ground. The two appear to overlap significantly only over a narrow range of Richardson numbers, which may be less than twofold.

1. THEORETICAL RELATIONS

When the ground is heated under calm conditions, and the temperature is not changing too rapidly, it has been shown (Priestley 1954) that the temperature profile in the convection-controlled layer and the accompanying eddy heat flux are determinate in terms of the lapse rate $-\partial\theta/\partial z$ of potential temperature at a single specified height z. The convective heat flux H then obeys the relation

$$\frac{H}{\rho\, c_p\, (g/\theta)^{1/2}\, |\partial\theta/\partial z|^{3/2}\, z^2} = \text{const.}$$

This will be written

$$\overset{*}{H} = \text{const.} \qquad . \quad . \quad . \quad . \quad (1)$$

where $\overset{*}{H}$ will be used throughout to denote the dimensionless number on the left-hand side above and the other symbols have their usual meaning.

When a wind is blowing at sufficient strength it must exercise some measure of control on the size and structure of the buoyant elements, and will induce additional vertical motions of a more purely dynamic origin. It is thought that both these effects arise primarily from wind shear $\partial u/\partial z$ rather than from wind velocity *per se* and that the macro-condition can then be specified, to a sufficient degree for present purposes, by recognizing the former quantity as an additional independent variable. The wind shear may be cast into dimensionless form, without introducing any further physical quantities, by use of the Richardson number, Ri, and with the problem thus reduced to one involving two dimensionless numbers, the heat-flux relation must take the form

$$\overset{*}{H} = f(Ri) \qquad . \quad . \quad . \quad . \quad (2)$$

where f is a function which remains to be discovered. This conclusion may be reached more rigorously by following the type of analysis given by Batchelor (1953).

Although (1) is the limiting form of (2) strictly derived only for $-Ri = \infty$, it may be expected to prove effectively valid beyond some finite value of $-Ri$ which may be determined by experiment. At the other extreme, for values of $-Ri$ sufficiently close to zero, the form of (2) will be that applicable to fully forced convection. Buoyancy then ceases to affect the motion or the heat-transfer coefficient, and the dynamically generated turbulence is taken to be the only agent in transferring heat down the temperature

gradient. The appropriate formulae are available from aerodynamic (mixing length) theory. By writing

$$H = -\rho c_p K_H \left(\frac{\partial \theta}{\partial z}\right)$$

we define an eddy conductivity which is equated to the eddy viscosity K_M, given by (see e.g., Sutton 1953, p. 318)

$$K_M = k^2 z^2 \left(\frac{\partial u}{\partial z}\right),$$

k denoting von Kármán's constant. From these it follows immediately that

$$\overset{*}{H} = k^2 |Ri|^{-1/2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(1) and (3) are simplest forms of the heat-flux relation in conditions of fully free and fully forced convection respectively, and it remains to determine their range of validity by appeal to experimental data. It should be repeated that formula (3) applies only to fully forced convection in its true sense, i.e., the value of k is that appropriate to neutral conditions. It is sometimes useful, for example in studies of momentum transport, to consider a more extended regime in which the above formula for K_M is used with a multiplying constant which is regarded as dependent on Ri. If K_H then behaved similarly to K_M it would imply, in the present context, a regime of quasi-forced convection having some though not all of the characteristics of true forced convection, but it is evident that the relation $\overset{*}{H} \propto |Ri|^{-1/2}$ would not then be valid. Quasi-forced convection in this sense is therefore here to be considered as a state belonging to the transition between (1) and (3).

2. Experimental results

Swinbank (1954) has recently reported a large number of sets of measurements of H, $\partial\theta/\partial z$, and $\partial u/\partial z$, which cover a wide range of lapse conditions. Excluding those about which some *a priori* doubt exists (as indicated by brackets in Swinbank's Table 1) and four* others, there remain 84 sets of measurements to form the material for analysis.

As might be expected from the nature of the measurements, etc., the values of $\overset{*}{H}$ versus Ri show large individual scatter; but when divided into groups according to the value of Ri and averaged, a fairly regular pattern emerges. Percentage errors are of more concern than absolute errors, and both $\overset{*}{H}$ and Ri represent multiplicative combinations of the quantities actually measured, so that averaging is most suitably carried out in terms of the logarithms. The results are given in Table 1 and displayed in Fig. 1 where $\overset{*}{H}$ is plotted against $-Ri$ on logarithmic scales. The value of Ri refers throughout to a height of 1·5 m.

TABLE 1. Determination of $\overset{*}{H}$ in terms of Ri

Range of $-Ri_{1\cdot5\,m}$	$<\cdot005$	$\geqslant\cdot005$ $<\cdot01$	$\geqslant\cdot01$ $<\cdot015$	$\geqslant\cdot015$ $<\cdot02$	$\geqslant\cdot02$ $<\cdot03$	$\geqslant\cdot03$ $<\cdot06$	$\geqslant\cdot06$ $<\cdot10$	$\geqslant\cdot10$ $<\cdot14$	$\geqslant\cdot14$
Number of measurements	3	9	11	7	14	11	10	11	8
Geometric mean $\lvert Ri\rvert$	0·0032	0·0071	0·0115	0·017	0·023	0·039	0·075	0·113	0·184
$\overline{\log \overset{*}{H}}$	0·45	0·10	0·00	− 0·04	− 0·19	− 0·18	− 0·05	− 0·185	− 0·23
Standard error of $\overline{\log \overset{*}{H}}$	0·17	0·08	0·07	0·10	0·10	0·08	0·10	0·065	0·10

* These comprise measurements made in the very early morning of 20 Feb. 1953 when the temperature was rising rapidly and the profile was marked by a reversal or pronounced discontinuity in gradient somewhat above the 2 m level. The principles on which the derivation of the above formulae have been based, that conditions are quasi-steady and the temperature profile adjusted to them, were not realized at this time.

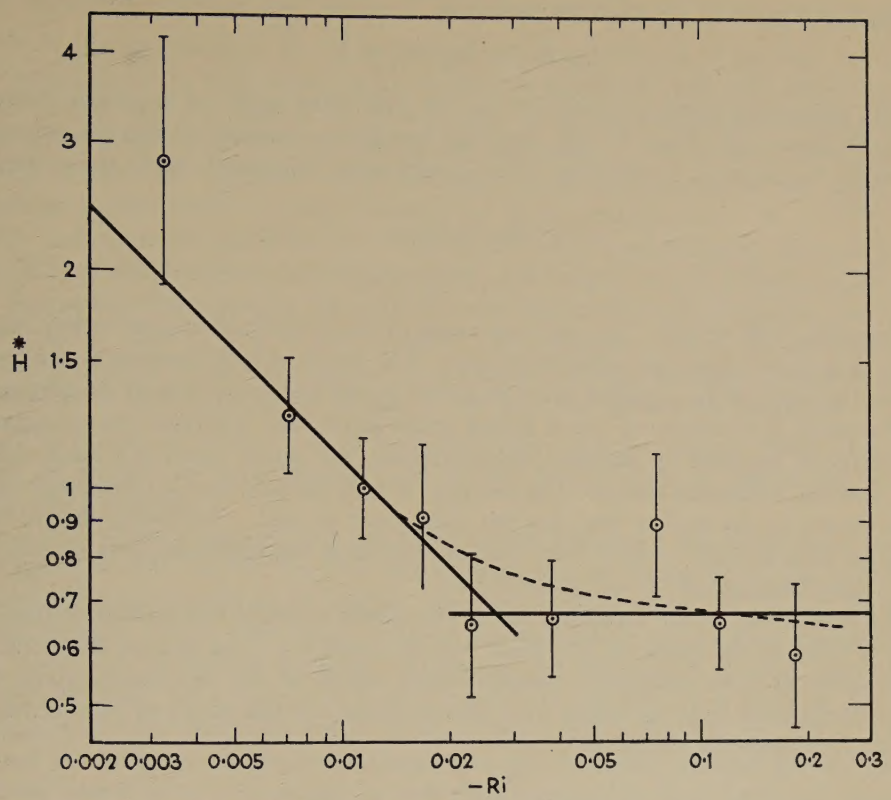

Figure 1. Dimensionless heat flux $\overset{*}{H}$ versus Richardson number (at 1.5 m). Mean values shown by circles and the limits of standard (log) error by short lines.

3. Fitting the data

The variables plotted are not independent combinations of the elements actually measured, since $\partial\theta/\partial z$ occurs in the numerator of one and the denominator of the other. The effect of errors in measurement of $\partial\theta/\partial z$ is therefore to introduce a spurious element of anti-correlation into the results, tending to turn the actual regression line towards one which has slope corresponding to a relation $\overset{*}{H} \propto |Ri|^{-3/2}$. The distortion depends on the accuracy of measurement of $\partial\theta/\partial z$ and is not likely to be serious in this case, except perhaps for the very small gradients which are relatively the most inaccurate.

The function f in (2) cannot be assigned any form at will, for the real physical condition must either be that of free or forced convection or of some intermediate state, progressing regularly with changing Ri from one extreme towards the other. The slope of the fitted curve should accordingly be restricted so that it shall nowhere lie outside the limits implied by (1) and (3) and shall be a monotonic function of Ri.

It may now be seen immediately that all the points up to $-Ri = 0{\cdot}03$ conform to a line of slope corresponding to (3). The full sloping line as drawn represents the best position of such a line calculated by the usual regression method. It is clear by inspection that better fit could be achieved only by a line which slopes more steeply, and this is excluded on physical grounds.

It is equally evident that the relation so obtained ceases to be valid for the larger values of $-Ri$. To determine the best overall fit for the right-hand portion of the

diagram, the regression equation was taken as

$$\log \overset{*}{H} = a \log |Ri| + b$$

and least squares calculation made on the 54 individual pairs of numbers extending from the largest $-Ri$ down to 0·02, keeping one group common to the two regression calculations to preserve continuity. The coefficients calculated, with their standard errors, were

$$a = 0{\cdot}00\ (\pm 0{\cdot}12)$$

$$b = -0{\cdot}17\ (\pm 0{\cdot}04).$$

Thus beyond $-Ri = 0{\cdot}02$, the extreme assumption of fully free convection fits the existing data better than any alternative law. The value of 0·68 (antilog $-0{\cdot}17$) then measures the value of the constant in (1) as shown by the horizontal line in the diagram.

Alternative procedures of curve fitting could have been followed but, recognizing the restrictions imposed by physical considerations, the result could not have differed greatly without worsening the fit. For example it may be held that a continuous curve, such as shown by the broken line, fits the data about as well. Accepting this it would still appear that beyond $-Ri = 0{\cdot}04$ free convection is the dominant process and that errors arising from use of (1) would be slight.

The chief doubt in the foregoing lies in the extent to which the picture of free convection is valid in the range of $|Ri|$ from 0·02 to about 0·1. An independent check is possible when, as from Swinbank's measurements, values of the root-mean-square temperature fluctuations $\sigma_T(z)$ at height z are also available, for the theory of free convection (Priestley 1954) predicts a relation

$$\sigma_T(z) \Big/ \left(\frac{\partial\theta}{\partial z}\right)_{1{\cdot}5\ \mathrm{m}} \propto z^{-1/3}$$

For $0{\cdot}02 < |Ri| < 0{\cdot}1$ there are available 19 values of σ_T at heights of 1·5 or 1·9 (average 1·7) m, and 14 at 23 or 29 (average 28) m. For these $\sigma_T \Big/ \left(\frac{\partial\theta}{\partial z}\right)_{1{\cdot}5\ \mathrm{m}}$ had the (geometric) mean values of 140 and 59 cm respectively, giving a ratio of 2·4 as compared with the theoretical $(28/1{\cdot}7)^{1/3} = 2{\cdot}5$.

4. Discussion

The implication is that when a lapse rate prevails in the lower atmosphere, the convective transfer of heat falls into one of two quite sharply differentiated regimes, those of fully forced and of dominantly free convection. The intermediate regime, which meteorologists have been inclined to regard as the typical state of affairs, appears to occupy only a rather narrow range of Richardson numbers. Judged by the diagram here presented, the transition appears to lie wholly within the range $0{\cdot}02 < -Ri < 0{\cdot}05$.

The position of the sloping straight line, as determined by regression, implies in (3) a value of 0·33 for von Kármán's constant. However, Swinbank's technique of eddy-heat flux measurement eliminates contributions from a pattern of eddies fixed in space as well as from those fluctuations whose periods are comparable with or longer than the period of measurement (5 min). It is to be expected that these losses become increasingly serious with increasing height, an effect confirmed in further analysis by Deacon (in press). The measurements can conveniently be divided into two groups, 47 sets where H was measured between 1·5 and 2 m, and 37 where it was measured at considerably greater heights, and the whole of the foregoing analysis has been repeated using only the former. The general features of the results, including the appearance of two quite

sharply separated regimes, are unaltered, but both the regression lines are displaced upwards to higher values of $\overset{*}{H}$. The revised constant value of $\overset{*}{H}$ for free convection is 0·81 and that for k, 0·37, which is in satisfactory agreement with the value found by Pasquill (1950), and by Deacon (in press) from Swinbank's stress measurements, and appears to be within about 10 per cent of the true value. The value of 0·81 may still be about 20 per cent too low owing to instrumental suppression of the most rapid fluctuations, a point which is under investigation at the present time.

The measurements analysed in this note are of a predominantly thermal nature (H and $\partial\theta/\partial z$) supplemented only by the value of the wind shear at a single height. No direct appeal has been made to the magnitude of the wind stress or friction velocity or to the detailed form of the velocity profile. This feature, backed by the agreement between the value of k and that obtained by the more orthodox approach, gives encouragement to believe that the general nature of the conclusions will be applicable to other sites. The numerical value of $\overset{*}{H}$ in free convection and the critical values of Ri may however be subject to some variation due to site factors such as degree and scale of thermal inhomogeneity.

It is not to be supposed that the same critical values of the Richardson number would apply to other levels, nor that the same regime of forced or of free convection will operate simultaneously at all heights. Accepting the argument of Deacon (in press) that the increase of $|Ri|$ with height implies an increase in the relative importance of purely convective as against dynamic turbulence, the implication of this paper is that there will normally be a regime of forced convection overlain by a layer of effectively free convection, the transition region between being only a shallow one. The level of transition will be lower the greater the lapse rate and the lower the wind speed. Deacon has given quantitative expression to this idea, suggesting that levels of transition between different regimes will be given by fixed multiples of the quantity

$$c_p\, T\, \tau^{3/2}/g\, \rho^{1/2}\, H$$

where τ is the shearing stress. The existence of a two-layer convection regime has already been suggested by Frankenberger (1952) and by Sutton (see published discussion of Frankenberger's paper) from quite other evidence.

Whereas any change in regime with height may be expected to be confirmed or otherwise from existing data, it is less easy to find data to verify the suggestion made here of a relatively abrupt change in regime with conditions at a given level. There is, however, supporting evidence in a study by Rider (1954), who measured shearing stress and evaporation and plotted these in dimensionless form against the Richardson number; both curves change from a concave to a convex relationship at values of Ri comparable with the critical values found here. Different attitudes and patterns of flight of soaring birds are undoubtedly related to different combinations of wind shear and lapse rate, but quantitative data on this aspect are lacking. Since Ri is proportional to $(\partial u/\partial z)^{-2}$, marked changes in convection pattern may at times accompany relatively slight changes in wind speed.

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Continuous convection from an isolated source of heat

By C. H. B. PRIESTLEY and F. K. BALL
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(Manuscript received 14 October 1954)

SUMMARY

Laws are derived for the variation of vertical velocity, w, and temperature, θ, with height, z, above a continuous circular source of heat of finite extent, on the assumptions that the lateral profiles are of similar shape at different distances above the source and that the resistance is a quadratic function of w. The lateral profiles are then taken to be Gaussian, though not all the properties deduced are dependent on this assumption. Complete solutions are obtained allowing for the thermal stratification of the environment; unless this stratification is neutral the common assumption that the vertical heat flux is constant with height is no longer true.

It is shown that the width of the column will increase uniformly with height. In general w increases with z at low elevations and then decreases, but this second stage does not occur when the environmental lapse rate is large and the source area sufficiently extensive. Under superadiabatic conditions the second stage, when it occurs, is followed by a third stage in which w increases again without limit; under subadiabatic conditions w decreases rather abruptly to zero at a certain ' ceiling ' height, for which a simple approximate formula is derived.

Numerical values illustrate the application of the solutions to problems of atmospheric pollution, frost prevention, and dry thermals. The solution for neutral conditions is compared with Railston's laboratory measurements.

1. Introduction

The problem to be discussed is the determination of the velocity and temperature fields above a continuous source of heat in a calm atmosphere. Attention will be confined to a circular source though the method adopted can be extended to a source which has the form of a long strip. Convection from a circular source has been considered previously by Schmidt (1941), Sutton (1950), and Rouse, Yih and Humphreys (1952). These authors each adopt a different approach, yet their treatments have certain limitations in common which it is the primary object of this work to remove.

Among the practical questions associated with the problem are those of atmospheric pollution, of orchard heating for frost protection, and of soaring flight, and these assume their greatest importance when the temperature stratification of the environment is either in a markedly stable or in a somewhat unstable condition. It has been shown that the behaviour of isolated buoyant bubbles is profoundly influenced by the temperature lapse rate of the environment, and the same must be true of continuous thermal columns though no formulation has so far been given. Previous treatments have been confined to conditions in which the atmosphere is in neutral equilibrium, and contain no suggestion of the means of generalization to other cases.* In particular they are based on the common assumption that the vertical heat flux is constant with height above the source; it will be shown hereunder that this may be deduced from more basic principles in neutral conditions, but it is not valid in the general case.

Again it is clear that, close above a heat source, motion is generated by buoyancy which outweighs the resistance to motion, and the air must therefore accelerate upwards initially and attain a maximum speed some distance above the source. The solutions previously obtained all demand that the velocity shall be finite or infinite at the source and decrease progressively upwards. This unrealistic feature arises because these solutions are not the most general ones, even in neutral conditions, and a more general solution has now been found which allows acceleration in the lower portion of the column.

* This omission has been rectified by Batchelor (1954), whose solutions however are still open in part to the objections outlined in the next paragraph.

The present analysis, like those referred to above, is based on the assumption that the rising air is in turbulent motion, the turbulence being generated by the motion in the column itself, and is confined to conditions of no wind so that the axis of the column is aligned with the force of buoyancy. The processes taken into account are the buoyancy of the column, its spreading and the entrainment of air into it, and the loss of momentum and heat by lateral diffusion. It has not been found possible to include radiation exchanges, though it is recognized that these must in practice play a part in restricting the ascent.

2. General treatment

The position in a circular-symmetric column will be defined by r, the distance from the axis, and z, the height above an origin which for the present is arbitrary. Consideration of the physical processes controlling the condition of a ring-shaped volume of air in the steady state yields an equation of continuity

$$\frac{\partial}{\partial z}(r\,w\,\rho) + \frac{\partial}{\partial r}(r\,u\,\rho) = 0 \qquad . \quad . \quad . \quad . \quad (1)$$

an equation of vertical motion

$$\frac{\partial}{\partial z}(r\,w^2\,\rho) + \frac{\partial}{\partial r}(r\,u\,w\,\rho) = r\,\frac{\theta'}{\theta_e}\,\rho\,g + \frac{\partial}{\partial r}(r\tau) \qquad . \quad . \quad (2)$$

and an equation of heat conservation

$$\frac{\partial}{\partial z}(r\,w\,\theta\,\rho) + \frac{\partial}{\partial r}(r\,u\,\theta\,\rho) = -\frac{1}{c_p}\frac{\partial}{\partial r}(rF) \quad . \quad . \quad . \quad (3)$$

where

w = vertical velocity

u = radial velocity

ρ = density

θ = potential temperature

θ_e = environmental potential temperature

θ' = excess potential temperature $(\theta - \theta_e)$

τ = vertical turbulent shearing stress

F = radial (potential) turbulent heat flux.

All quantities relate to mean values for the ring, and the vertical turbulent mixing has been neglected in comparison with the horizontal. The pressure is assumed to be undisturbed, and as is usual in problems of this type, the density is treated as constant except in so far as it affects the buoyancy.

From (1) and (2) may be derived the kinetic-energy equation

$$\frac{\partial}{\partial z}(\tfrac{1}{2}\,r\,w^3\,\rho) + \frac{\partial}{\partial r}(\tfrac{1}{2}\,r\,u\,w^2\,\rho) = r\,w\,\frac{\theta'}{\theta_e}\,\rho\,g + w\,\frac{\partial}{\partial r}(r\tau) \quad . \quad . \quad (4)$$

and from (1) and (3), with $\theta = \theta_e + \theta'$,

$$\frac{\partial}{\partial z}(r\,w\,\theta'\,\rho) + \frac{\partial}{\partial r}(r\,u\,\theta'\,\rho) = -\frac{1}{c_p}\frac{\partial}{\partial r}(rF) - r\,w\,\rho\,\frac{\partial\theta_e}{\partial z} \quad . \quad . \quad (5)$$

In order that the rate of entrainment of air into the column shall be finite, $r\rho u$ must remain finite as r tends to infinity, but at sufficient distance there will be no resistance

or lateral heat flow so that rF, $r\tau$, w, and θ' all tend to zero. Integration of (2), (4), and (5) from $r = 0$ to $r = \infty$ then gives

$$\frac{d}{dz}\int_0^\infty r\,w^2\,\rho\,dr = \int_0^\infty r\,\frac{\theta'}{\theta_e}\,\rho\,g\,dr \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{d}{dz}\int_0^\infty \tfrac{1}{2}\,r\,w^3\,\rho\,dr = \int_0^\infty r\,w\,\frac{\theta'}{\theta_e}\,\rho\,g\,dr - \int_0^\infty r\tau\,\frac{\partial w}{\partial r}\,dr \quad . \quad . \quad (7)$$

$$\frac{d}{dz}\int_0^\infty r\,w\,\theta'\,\rho\,dr = -\int_0^\infty r\,w\,\rho\,\frac{\partial \theta_e}{\partial z}\,dr \quad . \quad . \quad . \quad (8)$$

It is reasonable to assume, as has been done in treatments of kindred problems in turbulent diffusion, that the lateral profiles of w and θ' are of similar shape at different distances from the source. It is also assumed that the shearing stress is a quadratic function of the relative velocity. Writing then w_m, θ_m' for values on the axis of the column and R for some linear characteristic of its lateral extent, the similarity hypothesis is represented formally by equations

$$\frac{w}{w_m} = f\left(\frac{r}{R}\right), \quad \frac{\theta'}{\theta_m'} = h\left(\frac{r}{R}\right), \quad \frac{\tau}{\tfrac{1}{2}\,\rho\,w_m^2} = j\left(\frac{r}{R}\right)$$

and the equations (6) to (8) become

$$\frac{d}{dz}(R^2\,w_m^2) = a\,R^2\,\frac{\theta_m'}{\theta_e}\,g \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\frac{d}{dz}(R^2\,w_m^3) = b\,R^2\,w_m\,\frac{\theta_m'}{\theta_e}\,g - c\,R\,w_m^3 \quad . \quad . \quad . \quad (10)$$

$$\frac{d}{dz}(R^2\,w_m\,\theta_m') = -\,d\,R^2\,w_m\,\frac{\partial \theta_e}{\partial z} \quad . \quad . \quad . \quad . \quad (11)$$

where a, b, c, d are numbers, which will be referred to as *profile constants* since they are determinate in terms of the functions f, h and j.

The derivation has so far followed that of Rouse *et al.* (1952), save in the appearance of the term on the right-hand side of (11) which these authors have not considered. Schmidt's treatment is in many respects similar but his formulation contains no counterpart of (10), the alternative third equation being provided by the assumption that R is proportional to z. It is clearly preferable that this proportionality, if it is correct, should be deduced as a property of the solutions. Sutton also assumes the restricted form of (11), his remaining two equations being provided by invoking analogies with turbulent diffusion processes of other types.

In neutral conditions, $\partial\theta_e/\partial z = 0$, (11) integrates immediately to

$$R^2\,w_m\,\theta_m' = \text{constant } A \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where A is given in terms of the source strength Q by $\pi\rho\,c_p\,A \propto Q$, the constant of proportionality depending on the profile functions and being unity for the forms specified in the next paragraph. It may then be readily seen that there is a solution of the form $R \propto z$, $w_m \propto z^{-\frac{1}{3}}$, $\theta_m' \propto z^{-\frac{5}{3}}$, the constants of proportionality being fixed in terms of A and the profile constants. This is the solution, obtained by Schmidt and by Rouse *et al.*, which suffers from the limitation referred to in the Introduction. However it involves only two constants of integration, A and the unassigned origin of z, and is therefore not the general solution of (9), (10), and (12), since this must involve three such constants. This question will be discussed further in the next section.

In the general case, $\partial\theta_e/\partial z \neq 0$, no analytic solution has been found for arbitrary values of the profile constants, i.e. for arbitrary shapes of the lateral velocity and temperature profiles. In any case it appears that these shapes will be determined by the details of the turbulent mechanism, so that further information is required either about the detailed mechanism or more directly about the profile shapes. Following Sutton it may be argued that the profiles are Gaussian and the measures of dispersion are, approximately at least, the same for the two quantities w and θ, for which experimental confirmation in the laboratory has been provided by Rouse *et al.* and by Railston (1954). Taking then

$$f = h = \exp\left(-\frac{r^2}{2R^2}\right),$$

which fixes the scale of R, it is found that $a = d = 2$, $b = 3$, and the equations to be solved become

$$\frac{d}{dz}(R^2 w^2) = 2R^2 \frac{\theta'}{\theta_e} g \quad . \quad . \quad . \quad . \quad (13)$$

$$\frac{d}{dz}(R^2 w^3) = 3R^2 w \frac{\theta'}{\theta_e} g - c R w^3 \quad . \quad . \quad . \quad (14)$$

$$\frac{d}{dz}(R^2 w\theta') = -2R^2 w \frac{\partial\theta_e}{\partial z} \quad . \quad . \quad . \quad . \quad (15)$$

where it is understood that henceforward the concern will be only with the quantities w_m, θ_m' and R as functions of z, so that the suffix m can conveniently be dropped. The more general case in which the profiles are not exactly normal or equal is discussed in the Appendix.

No assumption has been made about the form of the function j, i.e. the value of c; to the extent that it is justifiable to assume that the relation between τ and w^2 is independent of the stability of the environment, c becomes a universal constant. Multiplying (13) by $3w$ and (14) by 2 and subtracting it follows immediately that

$$\frac{dR}{dz} = c \quad \text{or} \quad R = cz \; . \quad . \quad . \quad . \quad . \quad (16)$$

so that linear spreading of the column has been derived as a necessary property of the solution, and c may be regarded as a spreading coefficient. Eq. (16) should properly contain a constant of integration, which fixes the origin of z : the constant will here be absorbed by taking the origin at the point of concurrence obtained by producing the outline below the level of the finite real source. The height of the real source will be denoted by z_0.

3. Solution for neutral conditions

The general solution of (12), (13) and (14) is given by (16) together with

$$w = \left\{\frac{3Ag}{2\theta_e c^2 z} + \frac{B}{z^3}\right\}^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (17)$$

and

$$\theta' = \frac{A}{c^2 z^2}\left\{\frac{3Ag}{2\theta_e c^2 z} + \frac{B}{z^3}\right\}^{-\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (18)$$

B being the additional constant of integration. The solution may be viewed in one of two ways. c may be regarded as a known quantity, supplied by aerodynamic theory

following assumptions of the type made by Sutton (1950) about the nature of the turbulence; alternatively it may be determined from (16) by visual or other observations of the rate of spreading of the jet. A is known in terms of the strength of the source. The remaining quantity B is fixed by means of some boundary condition. For sources of different strength which satisfy the boundary condition $w = 0$, B will be proportional to A, so that $w \propto A^{\frac{1}{3}}$ and $\theta' \propto A^{\frac{2}{3}}$, properties which the present solution has in common with earlier ones and of which the latter has been verified in the laboratory by Railston (1954).

The boundary condition will most commonly take the form of assigning a value w_0 of w, or a value θ_0' of θ', at the source height z_0, which in turn is determined from (16) as R_0/c, where R_0 is the radius of the column at the source. The solution is then determinate in terms of the single boundary value, the source strength, and the spreading coefficient c. It is convenient to introduce a standard velocity w_* defined by

$$w_*^3 = \frac{3\,g\,A}{2\,\theta_e\,c^2\,z_0} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (19)$$

and to express the velocities in dimensionless form as $\overset{*}{w} = \frac{w}{w_*}$, $\overset{*}{w}_0 = \frac{w_0}{w_*}$. Introducing also a dimensionless height $\overset{*}{z} = \frac{z}{z_0}$, the solution (17) becomes

$$\overset{*}{w}^3 = \frac{1}{\overset{*}{z}} - \frac{1 - \overset{*}{w}_0{}^3}{\overset{*}{z}{}^3} \qquad . \qquad . \qquad . \qquad . \qquad (20)$$

It is then seen that the rising air will accelerate upwards initially provided that $\overset{*}{w}_0{}^3 < \frac{2}{3}$, attaining a maximum velocity given by

$$\overset{*}{w}{}^3 = \frac{2}{3^{\frac{3}{2}}}\{1 - \overset{*}{w}_0{}^3\}^{-\frac{1}{2}} \qquad . \qquad . \qquad . \qquad . \qquad (21)$$

at a height

$$\overset{*}{z} = \{3\,(1 - \overset{*}{w}_0{}^3)\}^{\frac{1}{2}} \qquad . \qquad . \qquad . \qquad . \qquad (22)$$

Thereafter the velocity decreases, and at sufficient height the first terms dominate the right-hand sides of (17) and (20) and the solution approaches the $w \propto z^{-\frac{1}{3}}$ law. If the velocity w_0 at the source is so great that $\overset{*}{w}_0{}^3 > \frac{2}{3}$ the accelerating stage will be absent, and the motion will decelerate throughout.

Corresponding solutions can be readily worked out for the case in which θ_0' rather than w_0 is given.

Whereas the general solution discussed above is relevant to the rise of smoke from factory chimneys, in a number of applications it may be sufficient to consider the special case where w_0 is taken to be zero at the source.* Such applications include convection from oil burners used for the heating of orchards and, were it permissible to neglect the effect of thermal stratification, the behaviour of natural thermal up-currents arising from differential heating of the ground. The results then take on a particularly simple form. The dimensionless velocity at any height is given by

$$\overset{*}{w}{}^3 = \frac{1}{\overset{*}{z}} - \frac{1}{\overset{*}{z}{}^3} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (23)$$

which is shown by the thick line in Fig. 1. The maximum velocity is equal to $0{\cdot}73\ w_*$, and this is attained at a height $z = z_0\,3^{\frac{1}{2}} = R_0\,3^{\frac{1}{2}}/c$, which would be at $7{\cdot}3\ R_0$ above the real source for $c = 0{\cdot}1$, the round number most appropriate (see Section 5).

* The following results and those of the next section apply more generally if z_0 is defined as the height, above the fictitious point source, at which w vanishes, and not necessarily identified as the real source.

The temperature excess θ' may be expressed in dimensionless form by writing $\overset{*}{\theta} = \theta'/\theta_*$, θ_* being a standard defined by

$$\theta_*^3 = \frac{2\,\theta_e\,A^2}{3\,g\,c^4\,z_0^5} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The solution corresponding to (23) is then

$$\overset{*}{\theta} = \frac{1}{\overset{*}{z}^2}\left\{\frac{1}{\overset{*}{z}} - \frac{1}{\overset{*}{z}^3}\right\}^{-\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (25)$$

which has also been calculated and plotted as the thick line in Fig. 2.

4. The effect of thermal stratification

No general analytical solution has been found for other than neutral conditions but in any given thermal stratification the solution can be found numerically. We shall be concerned here with the effect of thermal stratification *per se*, and so shall consider the condition in which $\partial\theta_e/\partial z$ is independent of height.

It is required to find the solution of (13), (14) and (15), the integral (16) having already been derived from the first two. The condition (12) no longer applies, and the first step is to find the integral which corresponds thereto in the more general case. Multiplying (15) by $R^2\,w\,\theta'$ and (13) by $R^2\,w^2\,\frac{\theta_e}{g}\,\frac{\partial\theta_e}{\partial z}$ and noting that $\theta_e\,\frac{\partial\theta_e}{\partial z}$ is effectively constant with height, the generalization of (12) follows as

$$R^2\,w\left(\theta'^2 + \frac{\theta_e}{g}\,\frac{\partial\theta_e}{\partial z}\,w^2\right)^{\frac{1}{2}} = \text{constant } A \quad . \quad . \quad . \quad (26)$$

If, close to the source, the velocity is zero or sufficiently small, so that the first term in the brackets is dominant, it remains true that $\pi\rho\,c_p\,A = Q$, so that A is uniquely related to the source strength; but otherwise this is not so. It is convenient to retain the symbol A for the more general quantity defined by (26), and the definitions (19) and (24) made in terms of it. It is seen that the property of constancy of flux with height belongs now not to the heat but to the quantity* measured by

$$\xi = \left(\theta'^2 + \frac{\theta_e}{g}\,\frac{\partial\theta_e}{\partial z}\,w^2\right)^{\frac{1}{2}}$$

By means of (16) and (26), R and θ' may now be eliminated from (13) and the equation for w in terms of z derived in the form

$$(z\,w)^2\,\frac{d}{dz}\,(z\,w) = \frac{g\,z}{\theta_e\,c^2}\left(A^2 - \frac{\theta_e}{g}\,\frac{\partial\theta_e}{\partial z}\,c^4\,z^4\,w^4\right)^{\frac{1}{2}} \quad . \quad . \quad (27)$$

This is more simply written in terms of the dimensionless quantities and a dimensionless measure of thermal stratification defined by

$$\gamma^4 = \frac{3}{2}\,\frac{z_0}{\theta_*}\left|\frac{\partial\theta_e}{\partial z}\right| \quad . \quad . \quad . \quad . \quad . \quad (28)$$

the equation becoming

$$\frac{d}{d\overset{*}{z}}\,(\overset{*}{z}\,\overset{*}{w})^3 = 2\,\overset{*}{z}\,\{1 \pm (\gamma\,\overset{*}{z}\,\overset{*}{w})^4\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (29)$$

* It may be shown that an element starting from rest with temperature excess ξ and accelerated by buoyancy without mixing will have temperature excess θ' on attaining a velocity w.

the positive or negative sign applying according as the lapse rate of temperature is super- or subadiabatic respectively. When w is zero at the source, which is the only case which will be considered in detail, the behaviour near the source is independent of the stratification of the environment, but at the greater heights the latter becomes critical.

(*a*) *Stable stratification*

When $\partial\theta_e/\partial z$ is positive, the substitution

$$\sin^{\frac{1}{2}} \Phi = \gamma \overset{*}{z} \overset{*}{w}$$

allows (29) to be expressed in the form

$$\sin^{\frac{1}{2}} \Phi \frac{d\Phi}{d\overset{*}{z}} = \tfrac{4}{3} \gamma^3 \overset{*}{z}$$

whence, since w is zero and hence Φ is zero at $\overset{*}{z} = 1$,

$$\int_0^{\Phi} \sin^{\frac{1}{2}} \phi \, d\phi = \tfrac{2}{3} \gamma^3 (\overset{*}{z}^2 - 1) \quad . \quad . \quad . \quad (30)$$

a form readily amenable to calculation. The curves in Fig. 1 show $\overset{*}{w} = \dfrac{w}{w_*}$ as a function of $\overset{*}{z} = \dfrac{z}{z_0}$ for a range of values of γ. For ease in assessing the effects in Fig. 1 of changes in lapse rate, or in characteristics of the source, it should be noted from (28), (24) and (16) that

$$\gamma \propto c^{\frac{1}{3}} z_0^{\frac{2}{3}} A^{-\frac{1}{6}} \left|\frac{\partial\theta_e}{\partial z}\right|^{\frac{1}{4}} \propto c^{-\frac{1}{3}} R_0^{\frac{2}{3}} A^{-\frac{1}{6}} \left|\frac{\partial\theta_e}{\partial z}\right|^{\frac{1}{4}}.$$

Thus a small range in γ covers a very wide range in source strength.

The most important new feature which emerges is the existence of a level, given by the intercept on the $\overset{*}{z}$ axis, at which the velocity vanishes and which therefore represents a ceiling beyond which the column does not penetrate. It is seen that, particularly for the smaller values of γ, the behaviour of the column will be similar to that under neutral conditions over the major portion of its life, the final disappearance of the velocity being rapid.

Even for the larger values of γ the curves do not deviate greatly up to the level of maximum velocity which, bearing in mind that $\gamma \propto \left(\dfrac{\partial\theta_e}{\partial z}\right)^{\frac{1}{4}}$, shows that the behaviour is insensitive to inversion strength up to this stage. These are fortunate circumstances in the study of sources located at ground level, for the strong curvature in the temperature profile very close to the ground can exert at most a minor influence and the assumption of the linear profile can be applied to the practical problem without serious error.

It may be noted also that the ascent of the curves to their maximum velocity is so steep that, for purposes of approximate calculation, an initial non-zero value of w at the source may be neglected so long as it does not exceed about one-half the maximum value indicated in the diagram; this may be verified by actual calculation.

Some quantitative values are of interest for illustration. With $\partial\theta_e/\partial z = 2 \times 10^{-3}$ °C/cm and $c = 0{\cdot}1$, consider first a single small oil burner of the lard-pail type used for orchard heating, of 10 in. diameter and producing heat at a rate of about 10 kw. For these values $z_0 = 125$ cm, $w_* = 220$ cm/sec, $\gamma = 0{\cdot}26$, whence a maximum velocity of 160 cm/sec would be reached at 90 cm above the ground with a ceiling height at about 50 ft (15 m). For a crude estimate of a typical source of atmospheric pollution, neglecting

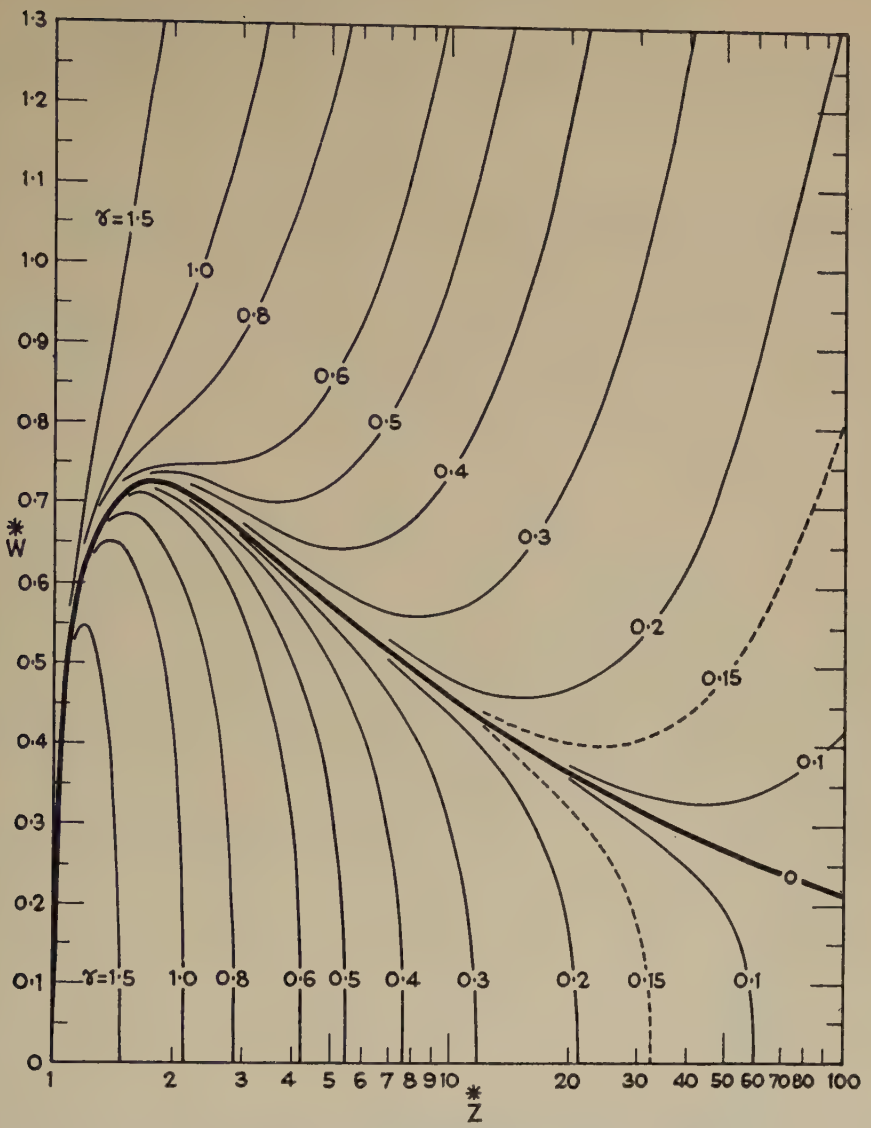

Figure 1. Velocity above a source with $w_* = 0$. Curves below and above the neutral curve refer to stable and unstable stratification respectively.

the velocity of emission, we might take a strength (Sutton 1950) of 5×10^7 BTU hr^{-1} or $1{\cdot}7 \times 10^4$ kw, and $R_0 = 1$ m. With the same inversion strength and spreading coefficient, $z_0 = 10^3$ cm, $w_* = 13$ m/sec, $\gamma = 0{\cdot}31$, so that a maximum velocity of 9·5 m/sec is predicted at 7 m, the ceiling at about 100 m, above the source.

A relatively simple approximate analytical expression for the ceiling height z_c may be derived from equation (30). w becomes zero when $\Phi = \pi$, whence

$$\gamma^3 \left(\frac{{z_c}^2}{{z_0}^2} - 1 \right) = \tfrac{3}{2} \int_0^{\pi} \sin^{\frac{1}{2}} \phi \, d\phi = 3{\cdot}60$$

or, transposing,

$$\left(\frac{z_c}{z_0} \right)^2 = \frac{3{\cdot}6}{\gamma^3} + 1 \simeq \frac{3{\cdot}6}{\gamma^3}$$

the approximation being made since it appears from the preceding calculations that γ

will be appreciably less than unity in most practical applications. Substituting for γ from (28) and then for θ_* from (24), the ceiling height is obtained in terms of the basic quantities as

$$z_c = 1{\cdot}55\, c^{-\frac{1}{2}} \left(\frac{g}{\theta_e}\frac{\partial\theta_e}{\partial z}\right)^{-\frac{3}{8}} \left(\frac{Ag}{\theta_e}\right)^{\frac{1}{4}} \quad . \quad . \quad . \quad (31)$$

It should be noted that z_0 and R_0 do not appear in this expression; under given environmental conditions the ceiling height will depend on the strength of the source but will, to a fair approximation, be independent of its size.

Having calculated w or $\overset{*}{w}$, the corresponding values for θ' or $\overset{*}{\theta}$ can be obtained most easily from (26); these are shown in Fig. 2. The reason for the abrupt decline in w as the ceiling height is approached is now seen to lie in a reversal of the buoyancy The height at which θ' changes sign is given by $\gamma \overset{*}{z}\overset{*}{w} = 1$ or $\Phi = \frac{\pi}{2}$, and so may be evaluated from (30) by

$$\left(\frac{z}{z_0}\right)^2 = 1{\cdot}8\,\gamma^{-3} + 1$$

or, in terms of ceiling height, by $z = 0{\cdot}71\, z_c$ for sufficiently small values of γ. At the ceiling height, the theory predicts that θ' shall decrease abruptly to $-\infty$; this is clearly unrealistic and in practice will be prevented by the operation of vertical diffusion. Some overshooting of the static equilibrium level may be a reality, however, and can sometimes be observed with smoke from chimneys when there is a light drift to incline the column slightly from the vertical. The possible existence of a zone of cooling may also have important implications in frost-prevention practice.

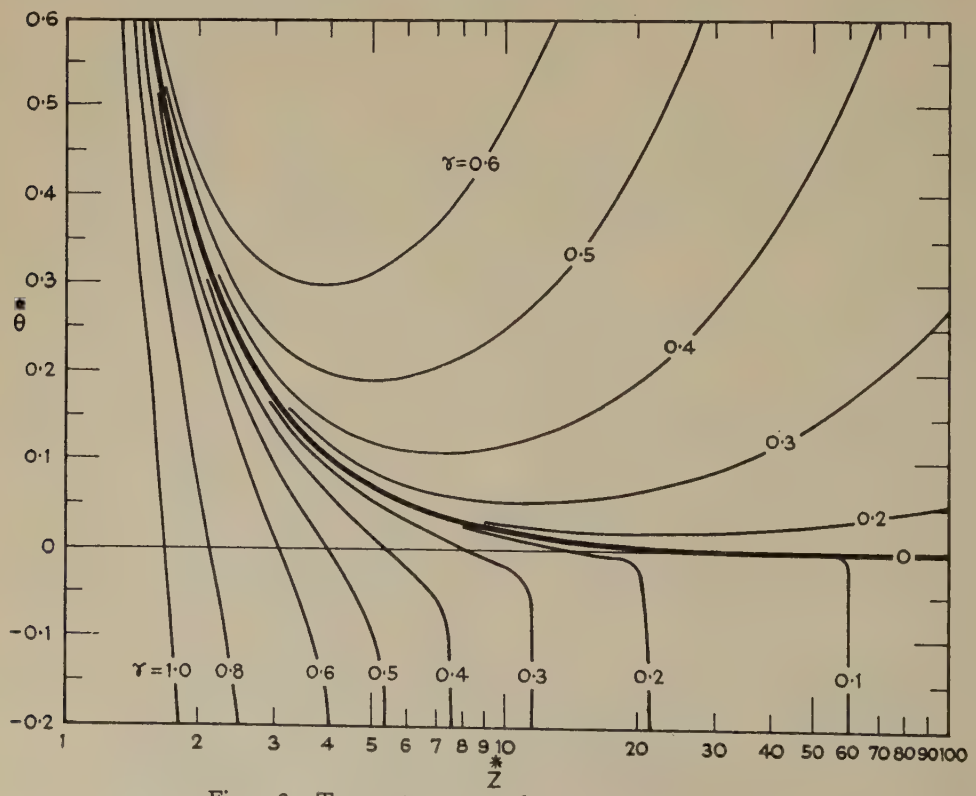

Figure 2. Temperature excess above a source with $w_0 = 0$.

(b) *Unstable stratification*

When $\partial\theta_e/\partial z$ is negative the substitution

$$\sinh^{\frac{1}{2}} \Phi = \gamma \overset{*}{z} \overset{*}{w}$$

leads to a hyperbolic relation in place of (30), which allows numerical solutions to be obtained in a similar manner. The results are also displayed in Fig. 1 and corresponding values of $\overset{*}{\theta}$ in Fig. 2. While for sufficiently small values of γ the form of the neutral solution is again closely followed for some way, eventually the velocity will increase indefinitely however small γ may be.

In equation (27) the second term within the bracket will ultimately dominate, and from this it may readily be derived that

$$w \simeq \tfrac{1}{2}\left(\frac{g}{\theta_e}\left|\frac{\partial\theta_e}{\partial z}\right|\right)^{\frac{1}{2}} z \quad . \quad . \quad . \quad . \quad (32)$$

at sufficiently large heights. It is of interest to compare this result with that for a single parcel ascending freely, i.e., which mixes in no way with its environment while subject to the buoyant acceleration. Whatever the initial temperature excess of such a parcel the temperature excess will ultimately be closely equal to $z\left|\frac{\partial\theta_e}{\partial z}\right|$ so that

$$w \simeq \left(\frac{g}{\theta_e}\left|\frac{\partial\theta_e}{\partial z}\right|\right)^{\frac{1}{2}} z$$

The ultimate rate of ascent of the column is therefore one-half that of the freely ascending parcel. These ultimate forms of the solution may not, of course, be realized in practice owing to the limited depth of superadiabatic layers.

The parallel problem of the parcel provides an explanation of an interesting property of the solutions shown in Fig. 1; for the smaller values of γ the column rather rapidly attains a fair velocity but over a range of higher levels the velocity declines before accelerating again into the final stage. For parcels of fixed size ascending with mixing through a superadiabatic environment it has been shown (Priestley 1953) that two distinct modes of behaviour result according as the parcel does or does not exceed a certain critical size; the smaller parcels are eventually brought to rest while the larger ones accelerate indefinitely. Whilst the law of mixing assumed in that work is not the same as that adopted here, the nature of the results are likely to be parallel. Since the present solutions have the property that the lateral extent of the column increases with height at a constant rate, the character of the motion is to be regarded as passing over at some stage from that corresponding to the smaller parcels to that of the larger ones. It may be noted, too, that $\gamma \propto R_0^{\frac{3}{2}}$: for sufficiently large γ, therefore, the decelerating stage would be expected to be absent, and Fig. 1 shows that this is so. The analogy between the parcel and the steady column, however, must only be invoked with caution, and an example is given below where it would be definitely misleading.

The dominant importance, under conditions of unstable stratification, of the horizontal extent of the column may be further illustrated by a numerical example. Taking 10^4 kw as representing a typical heat exhaust from an industrial chimney with $R_0 = 100$ cm, $c = 0{\cdot}1$, and $\left|\frac{\partial\theta_e}{\partial z}\right| = 10^{-5}$ °C/cm or one-tenth of the adiabatic lapse rate, then $\gamma = 0{\cdot}05$. This may be compared with the same total source of heat spread over a field of 200 m radius giving a source per unit area of 8 mw/cm^2 which should be representative, at least in its order of magnitude, of the *differential* rate above bare soil as compared with pasture when heated by the sun. γ now becomes equal to 1·6 which, as seen from Fig. 1, quite changes the character of the solution.

This work is concerned with sources of finite area but, when in final preparation, Batchelor's (1954) dimensional treatment of the point source problem came to hand. Of particular interest is the conclusion that a column can be formed under superadiabatic conditions without a heat source. Putting $A = 0$ in (26) it follows that

$$\theta' = \left(-\frac{\theta_e}{g}\frac{\partial\theta_e}{\partial z}\right)^{\frac{1}{2}} w$$

and from (27) that

$$w = \tfrac{1}{2}\left(\frac{-g}{\theta_e}\frac{\partial\theta_e}{\partial z}\right)^{\frac{1}{2}}\left(z - \frac{z_0^2}{z}\right).$$

The ultimate behaviour is identical with (32) and is independent of source strength or size. Batchelor's solution represents the special case with $z_0 = 0$.

These are steady-state solutions and it has only been shown that a column, once it is established, can continue without a source. On the other hand in an already turbulent environment (Priestley 1953) a parcel will not be set in motion unless the disturbance to w or θ' is of finite intensity or is applied to a sufficiently large area, and this would appear to apply equally to the initiation of a column.

5. Comparison with experimental data

Although the features of the foregoing solutions agree, in a qualitative way, with what is believed to occur in practice, there are available no experimental data from the atmosphere whereby a quantitative test can be made. However, under neutral conditions experiments have been made in the laboratory, notably by Railston (1954), to which the theory should equally apply provided the heated column is turbulent; (if the flow were laminar the quadratic relation between τ and w, implied in assuming constancy of c, would not hold).

Railston made measurements of the temperature field above a small electrical heater-element enclosed in a large chamber. Using various power inputs he confirmed, to a close approximation, the law $\theta' \propto Q^{\frac{2}{3}}$ or $\theta' \propto A^{\frac{2}{3}}$ predicted by the present theory in common with earlier ones. The lateral extent r of the jet was measured as 7·3 cm at 20 cm above the source, 11·3 cm at 40 cm, and 15·0 cm at 60 cm. Whereas this was interpreted as confirming a law of the type R proportional to z raised to some power rather less than unity, the accuracy was not sufficient to discriminate between this and the linear law (16), since the latter can be made to agree within 1 mm at each of the levels. The transverse distribution of temperature was found to follow the Gaussian closely.

It remains therefore to test the theory against the absolute values of θ' and the law of decay with height along the axis of the column as measured by Railston. Since the dependence of θ' on A has been tested independently, only one value of the power output (the middle, 9·92 watt, being the most suitable) need be considered for the present purpose. The lateral extent r quoted above does not correspond to R, but the two should be in fixed proportion if the distribution is truly Gaussian. R is the distance from the axis at which the temperature excess reaches 0·61 of the axial value; this occurs on the steepest part of the curve and can be read from Railston's Fig. 4, which gives $R = 3{\cdot}2$, 5·0, 6·8 cm at 20, 40, 60 cm above the source. From these we derive $c = 0{\cdot}09$, $z_0 = 15{\cdot}6$ cm. The theoretical values of θ' can now be calculated from (18) or from the simpler form (25) if it is assumed that $w = 0$ and hence $\theta' = \infty$ at the source; little difference is made by assigning a more realistic source value θ_0' in the full equation (18), e.g., $\theta_0' = 100°\text{C}$ reduces the calculated values at 5 and 10 cm by 0·7°C and 0·2°C respectively. Table 1 provides the comparison between temperatures measured and those calculated from the assumption that all the energy supplied is lost by convection.

TABLE 1. TEMPERATURE EXCESS OBSERVED AND CALCULATED

Distance above source (cm)	5	10	15	25	35	45	55	65	70	
Calculated value (°C)	24·0	14·3	9·5	5·9	4·1	3·0	2·4	1·8	1·65	
Observed value (°C)	12·9	7·7	5·8	3·4	2·3	1·7	1·3	1·0	0·9	
Ratio	0·54	0·54	0·61	0·58	0·56	0·57	0·54	0·56	0·55	0·56 (average)

It is clear that the law of decay has been predicted successfully over the whole range, but the calculated values are consistently too high by a factor of about 75 per cent. There must be some loss of heat by radiation from the element and a further loss by conduction through the leads and supports; also, under the conditions of the experiment, it is probable that there was a slight induced upward draught throughout the chamber (the chimney effect), driven by energy drawn from the heater. Whilst these effects may be individually rather small they combine in multiplicative fashion so that the true ' convection-efficiency ' of the heater may be substantially less than 100 per cent. It is thought unlikely, however, that the whole of the discrepancy can be attributed to losses of this nature.

The remainder is most likely due to a recognized shortcoming in the theory itself which is hard to take into account in any formal manner. When observing smoke columns in the open it may be seen that, even on a calm day, the ascent is markedly irregular; well-defined individual surges may be seen to ascend through the column at speeds much in excess of the average, and it is improbable that this effect can be avoided in the laboratory however careful the experimental technique. The faster moving puffs are presumably associated with higher than average temperatures. Relations such as (12) should therefore more accurately be written

$$R^2 \left(\overline{w}\, \overline{\theta'} + \overline{w'\, \theta'} \right) = A,$$

the usual convention of turbulence being employed whereby the quantities are split into their average and variable components. So long as mean values alone are considered the effective source strength should be reduced accordingly, or conversely, use of the theory without allowing for this form of vertical turbulent diffusion must result in values for $\overline{w}$ and $\overline{\theta'}$ which are too high. If the two terms within the brackets are in the same ratio at all heights, which in the absence of better knowledge would seem to be a reasonable working hypothesis, the results of the theory remain unaltered in so far as they relate to functional form.

6. REVIEW

The strength of the present formulation lies in its relative freedom from adjustable constants or semi-empirical points of appeal. The disposable quantities reduce to a single one, the spreading coefficient c, which there is reason to believe may approach the character of a universal constant. This and the assumption of the Gaussian transverse distribution, for which there is good laboratory evidence, represent the only features not deducible from first principles or known boundary conditions. Though it appears, at first, remarkable that so much has been deduced without forming any detailed picture of the mechanisms of entrainment and diffusion, it must be recalled that these are implicit in the form of the profile functions f and g.

At more than one stage the neglect of the vertical diffusion of heat has been recognized as a serious limitation to this as to previous theories, and it is felt that if a satisfactory means can be found of allowing for this effect a substantially complete theory will be available.

The paper has been concerned with the pattern of convection as such, and the practical application to the estimation of the concentration of smoke, etc., from factory chimneys is somewhat outside its scope, but one very simple result follows immediately. If the settling velocity can be neglected, the concentration x on the axis above a source Q_P of pollution is given by

$$\pi R^2 w x = Q_P,$$

so that x behaves like the quantity ξ defined in Section 4 and may be obtained from the calculated values of w and θ'. This assumes that the horizontal distribution of x is the same as that of w and θ'.

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Appendix

Profiles not exactly normal or equal

It will have been observed that the solution (16), and all the other results which are consequent thereon, can be derived only when the profile constants a and b are in the ratio 2 to 3, as when the lateral profiles of w and θ' are both normal and of equal dispersion. Measurements from this and other related flow patterns indicate that these assumptions are true approximately but not exactly, and it is necessary to examine the extent to which the solutions continue to apply in these circumstances.

Retaining the similarity assumptions preceding Eq. (9), but equating the functions f and h only approximately with $\exp(-r^2/2R^2)$ and with each other, leads to the result that $b = 3(1 + \epsilon)$, $a = 2(1 + \epsilon')$, $d = 2(1 + \epsilon'')$, where the corrections ϵ are supposed small compared with unity. The correction to d can be absorbed in (15), retaining the factor 2, by using an appropriately adjusted environmental lapse rate; similarly the corrections to a or b, *but not both*, can be absorbed in (13) or (14) by regarding the flow as taking place with an adjusted value of gravity g or of temperature θ_e. Thus the problem of approximation may be treated quite generally by retaining (13) and (15), with (14) in a modified form

$$\frac{d}{dz}(R^2 w^3) = 3(1 + \epsilon) R^2 w \frac{\theta'}{\theta_e} g - c R w^3 \quad . \quad . \quad (14a)$$

From (14a) and (13) may then be derived

$$\frac{dR}{dz} = c - 3\epsilon \frac{R \theta' g}{w^2 \theta_e} \quad . \quad . \quad . \quad . \quad (33)$$

The previous solutions will therefore be approximately valid provided they are applied exclusively to the range of variables for which $3 R \theta' g/(c w^2 \theta_e)$ remains of order less than $1/\epsilon$.

A somewhat closer approximation can be obtained in the form $R = c'z$ where $c' = c + O(\epsilon)$, and the quantities $\overset{*}{w}$, $\overset{*}{\theta}$, etc., are obtained as before with c' replacing c

in expressions such as (19) and (24). Use of these relations in (33) gives

$$c' = c - 2\epsilon \frac{\overset{*}{\theta}\overset{*}{z}}{\overset{*}{w}^2} c'$$

or

$$c' = c \Big/ \left(1 + \frac{2\epsilon \overset{*}{\theta}\overset{*}{z}}{\overset{*}{w}^2}\right) \quad . \quad . \quad . \quad . \quad (34)$$

Isopleths of $\overset{*}{\theta}\overset{*}{z}/\overset{*}{w}^2$ are shown in Fig. 3. As an illustration of their application it may be seen that, for $\epsilon = +0{\cdot}1$, the value of c'/c becomes 0·63 at $\overset{*}{z}$ from 1·25 to 1·35, depending on γ. For large values of γ in lapse conditions (strong lapse, extensive source) it subsequently undergoes only slight change. In most other cases likely to be of practical interest it increases to 0·77 at about the height of maximum velocity; thereafter under lapse conditions it decreases again slowly to its ultimate value of 0·63, in neutral conditions increases to 0·83, whilst under inversion conditions it increases to unity at the level where $\overset{*}{\theta} = 0$ and thereafter the approximate solution soon becomes invalid.

By taking therefore a constant value of c', equal to the average of the appropriate values above, the solution $R = c'z$, etc., provides an approximation over most of the range of physical interest. It ceases to be applicable very close to the source and very close to the ceiling height; these are in any case the points at which the solutions are already most open to doubt on account of the basic assumptions (undisturbed pressure, constancy of density *per se*, and neglect of vertical diffusion). This conveys one particular warning that in numerical application the linear dimension of the source, where this is known (e.g., with smoke pots or some laboratory experiments), must not be identified with the linear dimension of the column at the source.

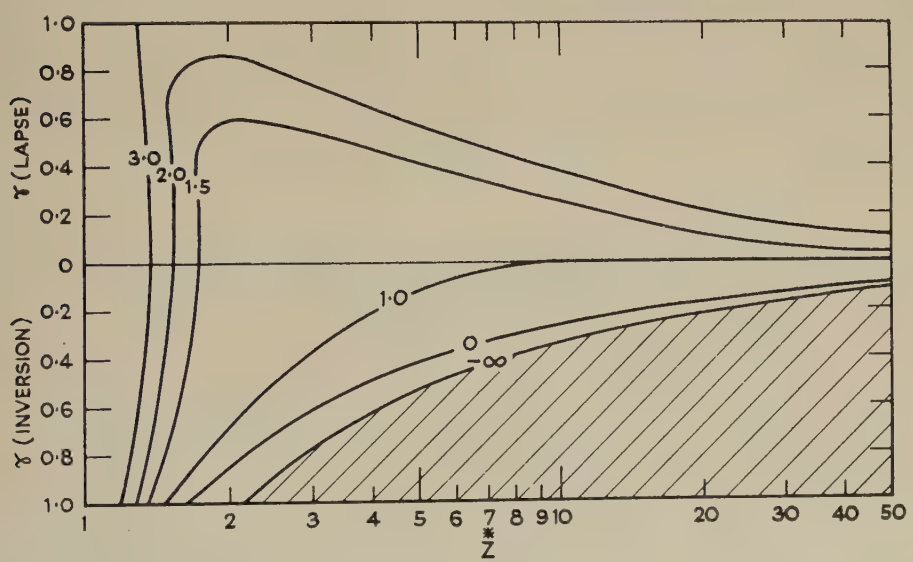

Figure 3. Isopleths of $\overset{*}{\theta}\overset{*}{z}/\overset{*}{w}^2$. Shaded region lies above the ceiling height.

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The frequency distribution of the annual maximum (or minimum) values of meteorological elements

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SUMMARY

It is shown that the probability P that the annual maximum value of a meteorological element is less than a given value x can be expressed as $\exp\left\{-\left(1-\frac{x}{a}\right)^{1/k}\right\}$ where a and k are constants such that ak is positive.

The parameters a and k can be evaluated from data of annual maximum values for a given period of years. Hence we derive the average highest and lowest in sets of T annual maxima, and also the maximum value to be expected once in T years.

1. INTRODUCTION

(a) *Outline of the problem and its solution*

Given the maximum values attained by a meteorological element in a relatively short period of years, we are often asked to predict the maximum value to be expected once in say 50 or 100 years. Examination of the data shows that, approximately, the maximum value attained increases linearly with the logarithm of the period; and Gumbel (1935) gives theoretical justification for this approximation. Defects in the approximation have been pointed out by Barricelli (1943) and by Brooks and Carruthers (1953). For temperatures the Gumbel approximation overestimates, and for rainfall underestimates, the maximum values reached in long periods.

Fisher and Tippett (1928) showed that extreme values satisfy a functional equation. The writer has found a general solution of this equation which describes all the types of distribution actually found in meteorological data. The Gumbel approximation is shown to be included in the general solution as a special case.

(b) *Definitions of symbols*

x_i $(i = 1, 2 \ldots N')$ = daily values of a meteorological element in a year of N' days
x = a general value of the element
$f(x)$ = the average annual number of daily values greater than x
$y = -\ln f(x)$
P = the probability that an annual maximum value is less than x
$X_1, X_2, \ldots X_n$ = a series of n annual maxima arranged in ascending order
μ = the modal value of the X's
ξ_T = the average of the greatest members in sets of T annual maxima
ξ_1 = the average annual maximum
σ_T = the standard deviation of the greatest members in sets of T annual maxima
σ_1 = the standard deviation of the annual maxima
σ_2 = the standard deviation of the greater members in pairs of annual maxima
dp = an element of probability
ξ_T' = the average of the least members in sets of T annual maxima

(c) *Frequency distribution of extreme values*

Of the daily values, x_i, of a meteorological element in a year, N may be considered independent and the probability of one of these daily values being less than x is $\{1 - f(x)/N\}$. Then the probability of all the daily values in a year being less than x, i.e., of the annual maximum being less than x, is $\{1 - f(x)/N\}^N$, and in T years $\{1 - f(x)/N\}^{NT}$; or, with sufficient exactness

$$\exp\{-f(x)\} \quad \text{and} \quad \exp\{-Tf(x)\} \quad \text{respectively} \qquad . \qquad . \qquad (1)$$

Now

$$f(x) = e^{-y} \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

and the probability that an annual maximum value is less than x is

$$P = \exp(-e^{-y}), \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

where P takes values from 0 to 1 as y takes values from $-\infty$ to $+\infty$. A table of corresponding values of P and y for $P = 0{\cdot}01\ (0{\cdot}01)\ 0{\cdot}99$ is given in Appendix 1.

If we arrange a series of n annual maxima in ascending order of magnitude $X_1, X_2 \ldots X_n$, we know, by considering values of x a little less than and a little greater than that of the mth value of the series, that the true cumulative frequency of the mth value is greater than $(m-1)/n$ and less than m/n. Engineers (Hazen 1930) have adopted a compromise frequency

$$P = (m - \tfrac{1}{2})/n = (2m-1)/2n \qquad . \qquad . \qquad . \qquad (4)$$

If the values of y corresponding to the values of $P = 1/2n, 3/2n, \ldots (2n-1)/2n$ are plotted against the values of $x = X_1, X_2 \ldots X_n$ then the points lie on a curve $y = y(x)$.

(d) *Gumbel's equation for the curve $y = y(x)$*

Fisher and Tippett (1928) and Gumbel (1935) proved that in the limit for large values of N the curve $y = y(x)$ is a straight line.

Gumbel derived the equation of this straight line in the form

$$y = (1/\beta)(x - \mu); \qquad . \qquad . \qquad . \qquad . \qquad (5)$$

μ is the modal value of the X's and $\beta = 0{\cdot}78\sigma_1$, where σ_1 is the standard deviation of the X's. Hence he derived the difference between ξ_T and ξ_1, the average maxima in T years and in one year respectively,

$$\xi_T - \xi_1 = \beta \ln T = 0{\cdot}78\,\sigma_1 \ln T \qquad . \qquad . \qquad . \qquad (6)$$

(e) *Barricelli's modification of Gumbel's theory*

Barricelli (1943) pointed out that Gumbel's theory does not always give satisfactory results when it is applied to meteorological data and modified the theory by allowing for variation in the annual number of daily values greater than x. His result has been written by Brooks and Carruthers (1953) in the form

$$(\xi_T - \xi_1)/\sigma_1 = 0{\cdot}78 \ln T\,(1 - 1{\cdot}4\,\lambda) + 2\lambda \qquad . \qquad . \qquad (7)$$

where $\lambda = 1 - (\sigma_2^2/\sigma_1^2)$ and varies in value from 0 (when the curve $y = y(x)$ is Gumbel's straight line) to 0·32 (when the maximum values are normally distributed). The theory does not allow for negative values of λ, that is for $\sigma_2^2 > \sigma_1^2$.

2. Suggested new frequency distribution

(a) *The types of y, x curve found in nature*

y is always an increasing function of x, but three types of y, x curve may be distinguished according to whether dy/dx increases or decreases with increasing x, or is constant. Gumbel's theory infers that dy/dx is constant, while Barricelli's theory infers that dy/dx is either constant or increasing with increasing x.

Figure 1 shows eight examples of the y, x curves for maximum values of certain meteorological elements. (a)-(c) are of Type I (dy/dx decreasing with increasing x), (d) is of Type II (dy/dx constant) and (e)-(h) are of Type III (dy/dx increasing with increasing x).

The rate of change of dy/dx in all the curves is small, justifying Gumbel's straight line as a first approximation.

As might be expected, Type III is the type most frequently found in nature, since it is reasonable to expect the maximum values to have an upper bound. Type II may be considered as the limiting curve of the set of Type III curves.

Maximum falls of rain have the Type I distribution. This is understandable, since to a considerable extent rainfall amounts are "uncontrolled," and extremely high falls may be recorded.

(b) *Suggested general equation for the curves*

A general equation for the y, x curves will now be suggested which includes all three of the Types I, II and III found in nature and is a general solution of the Fisher-Tippett (1928) functional equation satisfied by extreme values.

The proposed curve is

$$x = a\,(1 - e^{-ky}) \qquad . \quad . \quad . \quad . \quad (8)$$

where ak is positive.

$$dy/dx = (1/ak)\, e^{ky} \qquad . \quad . \quad . \quad . \quad (9)$$

When k is positive,

$$\left.\begin{array}{lll} y = -\infty & x = -\infty & dy/dx = 0 \\ y = 0 & x = 0 & dy/dx = 1/ak \\ y = +\infty & x = a & dy/dx = \infty \end{array}\right\} \quad . \quad . \quad (10)$$

When k is negative,

$$\left.\begin{array}{lll} y = -\infty & x = a & dy/dx = \infty \\ y = 0 & x = 0 & dy/dx = 1/ak \\ y = +\infty & x = +\infty & dy/dx = 0 \end{array}\right\} \quad . \quad . \quad (11)$$

When $k \to 0$

$$x \to aky \qquad . \quad . \quad . \quad . \quad . \quad (12)$$

Thus, when k is negative, equation (8) gives a curve of Type I; when k is positive, of Type III; and when k is zero, of Type II (Gumbel).

The Fisher-Tippett (1928) functional equation may be briefly described here. Since the upper member of a sample of ls members is the upper member of a sample of l upper members of samples of s, then if P is the probability of an observation being less than x, the distribution of upper members satisfies the functional equation

$$P^s(x) = P(a_s x + b_s) \qquad . \quad . \quad . \quad . \quad (13)$$

where a_s and b_s are functions of s. In our notation $P = \exp(-e^{-y})$.

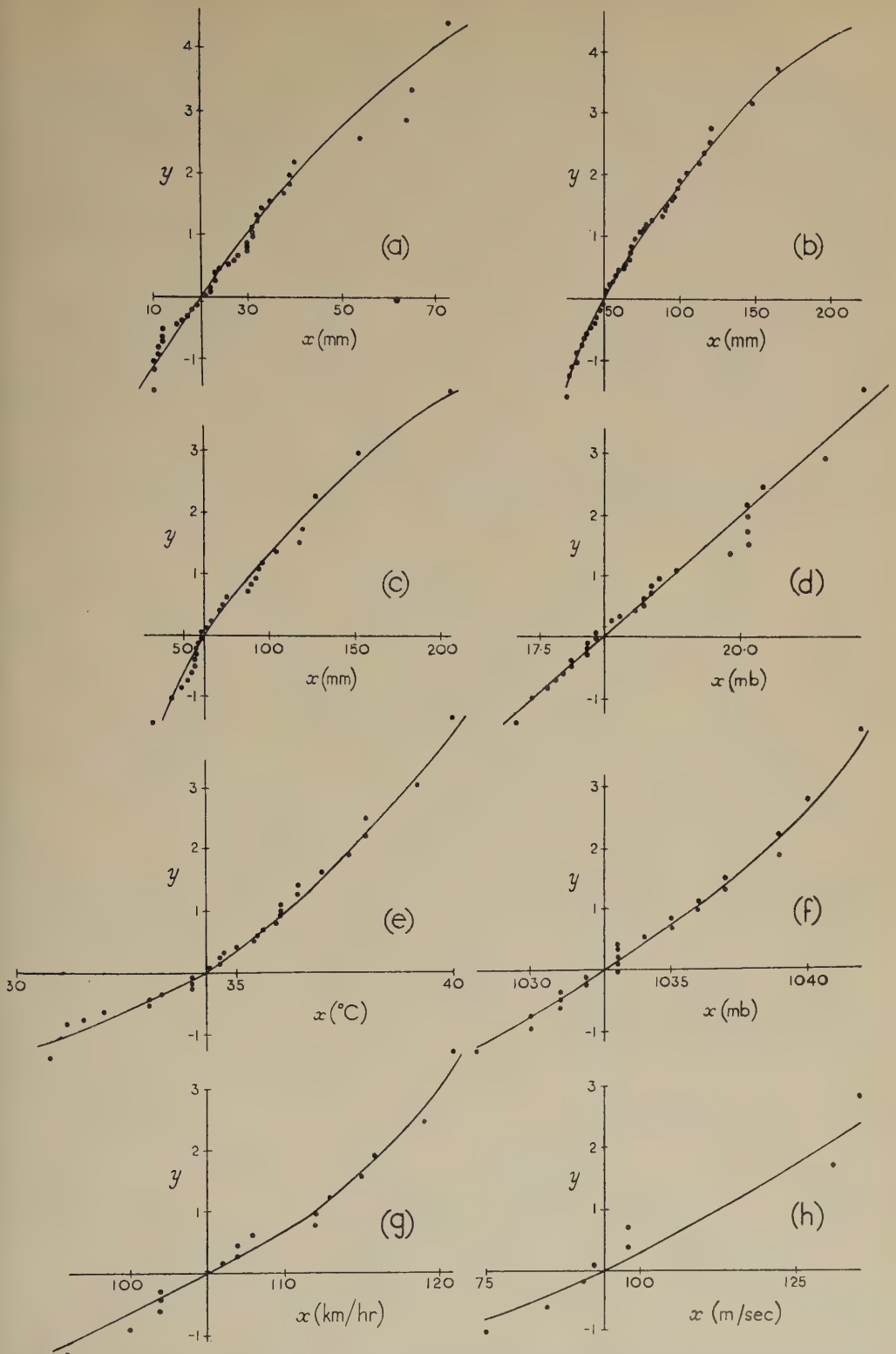

Figure 1. y, x graphs of distribution of maximum values. (a) Naples, annual maximum hourly rainfall, 1888-1933; (b) Marseilles, annual maximum daily rainfall, 1882-1946; (c) Tripoli, annual maximum four-day rainfall, 1919-1948; (d) Milan, annual maximum vapour pressure, 1921-1950; (e) Siros, maximum August temperature, 1898-1929; (f) Marseilles, annual maximum pressure, 1923-1946; (g) Trieste, annual maximum hourly wind speed, 1903-1920; (h) Tateno, monthly maximum wind speed at 9 km for Dec., Jan., Feb., 1950-1952.

Authorities for data : (a) Guerrieri, E. 1935, Rovesci e massima intensita di pioggia . . . a Capodimonte; (b), (f), *Bulletin annuel. . . . Bouches-du-Rhône*; (c) Fantoli, A. 1952, Le pioggie della Libia; (d) Campa, M. 1952, Valori medi ed estremi annuali . . . a Milano; (e) *Annales de l'Observatoire National d'Athènes*; (g) Polli, S. 1949, 100 anni di osservazioni . . . a Trieste; (h) Aerological data of Japan.

It can be shown (a proof is given in Appendix 3) that equation (8) is a general solution of the Fisher-Tippett functional equation (13). It has been shown in equations (10) to (12) that, for different values of k, equation (8) gives all the types of y, x curves found in nature, with Gumbel's straight line as a special case when $k \to 0$. For negative k, x is bounded below, which is true for maximum falls of rain; and for positive k, x is bounded above, which implies an upper bound to extreme values of wind speeds and temperatures. Examination of available data shows this to be reasonable.

(c) The average of the greatest members in series of T annual maxima

The probability distribution of the maximum values in periods of T years is, combining (1) and (2),

$$dp = d\{\exp(-Te^{-y})\} \quad . \quad . \quad . \quad . \quad (14)$$

Then ξ_T and σ_T, the average and standard deviation respectively of the greatest members in series of T annual maxima, are given by

$$\xi_T = \int_{y=-\infty}^{y=+\infty} x \; d\{\exp(-Te^{-y})\}$$
$$\sigma_T^2 + \xi_T^2 = \int_{y=-\infty}^{y=+\infty} x^2 \, d\{\exp(-Te^{-y})\} \quad . \quad . \quad . \quad (15)$$

Then
$$\xi_T = \int_{-\infty}^{\infty} a(1 - e^{-ky}) \, d\{\exp(-Te^{-y})\}$$

After making the transformation $v = Te^{-y}$ this integral can be readily evaluated as

$$\xi_T = a(1 - k\,!\,T^{-k}) \quad . \quad . \quad . \quad . \quad (16)$$

Therefore
$$\xi_T - \xi_1 = a \,.\, k\,!\,(1 - T^{-k}) \quad . \quad . \quad . \quad . \quad (17)$$

Similarly $\sigma_T^2 + \xi_T^2 = a^2\{1 - 2(k\,!)\,T^{-k} + (2k)\,!\,T^{-2k}\}$ and hence

$$\sigma_T^2 = a^2\,T^{-2k}\{(2k)\,! - (k\,!)^2\} \quad . \quad . \quad . \quad (18)$$

Then $\sigma_1/\sigma_T = T^k$ and in particular

$$\sigma_1/\sigma_2 = 2^k \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

From equation (18) we derive

$$a = \sigma_1\{(2k)\,! - (k\,!)^2\}^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (20)$$

Hence
$$\frac{\xi_T - \xi_1}{\sigma_1} = \frac{k\,!\,|\,1 - T^{-k}|}{\{(2k)\,! - (k\,!)^2\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (21)$$

When k is negative (Type I curve), $\sigma_1 < \sigma_2$

When k is zero (Type II curve), $\sigma_1 = \sigma_2$

When k is positive (Type III curve), $\sigma_1 > \sigma_2$

The evaluation of the ratio σ_1/σ_2 enables us to decide the type of curve, and also, by giving the value of k, to determine $(\xi_T - \xi_1)/\sigma_1$, from equation (21).

Now, for small k

$$k! = \exp(-\gamma k + \tfrac{1}{2} C_2 k^2 - \ldots) \quad . \quad . \quad . \quad (22)$$

Where $\gamma = 0{\cdot}5772\ldots$ and $C_2 = \pi^2/6 = 1{\cdot}6449\ldots$

Then, as $k \to 0$

$$\xi_T - \xi_1 \to 0{\cdot}78\,\sigma_1 \ln T \quad . \quad . \quad . \quad . \quad . \quad (23)$$

This is Gumbel's result.

The ratio $(\xi_T - \xi_1)/\sigma_1$ is given in Table 1 for $\sigma_1/\sigma_2 = 0{\cdot}85\,(0{\cdot}05)\,1{\cdot}20$ and for $T = 10$, 25, 50 and 100. The corresponding values for samples of size T from a normal population (Tippett 1925) are added.

TABLE 1. VALUES OF THE RATIO $(\xi_T - \xi_1)/\sigma_1$

σ_1/σ_2	T 10	25	50	100
0·85	1·85	2·90	3·87	5·01
0·90	1·85	2·80	3·61	4·50
0·95	1·84	2·66	3·33	4·02
1·00	1·79	2·50	3·05	3·59
1·05	1·73	2·33	2·80	3·21
1·10	1·67	2·18	2·57	2·89
1·15	1·61	2·06	2·37	2·62
1·20	1·56	1·95	2·20	2·40
Normal	1·54	1·98	2·25	2·51

(d) *The frequency distribution of annual minima*

If we take the direction of increasing x towards the lower values of recorded minima, no change is needed in any of the equations or in the description of the types of y, x curve.

Figure 2 shows two y, x curves for distributions of minimum values; they are of Type III.

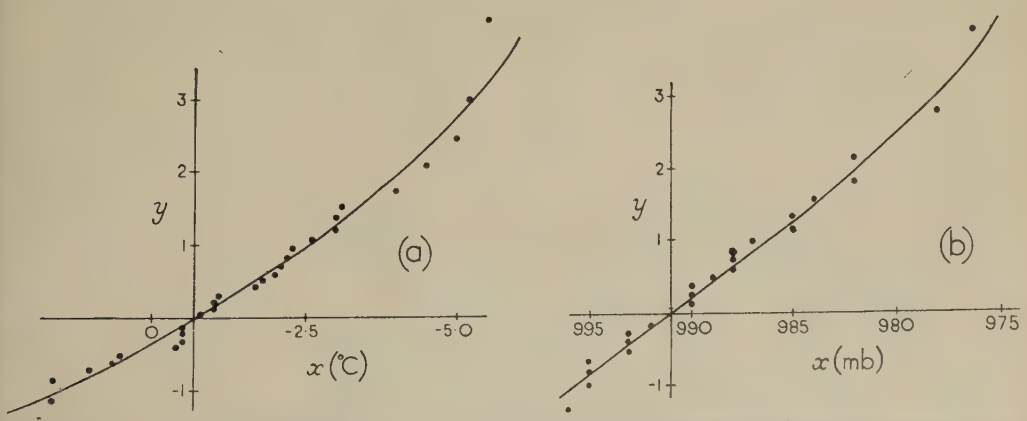

Figure 2. y, x graphs of distribution of minimum values. (a) Athens, annual minimum temperature, 1900-1929; (b) Tortosa, annual minimum pressure, 1910-1934.

Authorities for data: (a) Annales de l'Observatoire National d'Athènes; (b) Rodes, L. 1936, Estudio climatológico ... de Tortosa.

(e) *The frequency distribution of the least members of series of T annual maxima*

The probability of all the daily values in a year being greater than x is, from equation (1), equal to

$$1 - \exp\{-f(x)\} = 1 - \exp(-e^{-y})$$

and the probability of all the annual maxima in T years being greater than x is

$$\{1 - \exp(-e^{-y})\}^T. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The average ξ_T' of the least members in sets of T annual maxima is given by

$$\xi_T' = \int_{y=-\infty}^{y=+\infty} -x\,d\left[\{1 - \exp(-e^{-y})\}^T\right] \quad . \quad . \quad . \quad (25)$$

i.e.

$$\xi_T' = \int_{-\infty}^{\infty} -a(1 - e^{-ky})\,d\left[\{1 - \exp(-e^{-y})\}^T\right]$$

which, after expanding $\{1 - \exp(-e^{-y})\}^T$ and making the transformation $v = e^{-y}$ can be evaluated as

$$a\{k\,!\,S(T, k) - 1\} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where

$$S(T, k) = \sum_{r=1}^{T} (-1)^{r+1} \frac{T\,!}{r\,!\,(T-r)\,!} \cdot r^{-k}$$

It can be shown that $S(T, k)$ is identically equal to 0 for $k = -1$, to 1 for $k = 0$, to $\left(1 + \frac{1}{2} + \ldots \frac{1}{T}\right)$ for $k = 1$, and to $1 + \frac{1}{2}(1 + \frac{1}{2}) + \ldots \frac{1}{T}\left(1 + \frac{1}{2} + \ldots \frac{1}{T}\right)$ for $k = 2$; and that for large T,

$$\left.\begin{aligned} k\,!\,S(T, k) &\simeq \{S(T, 1)\}^k + \tfrac{1}{2}k(k-1)\{S(T, 1)\}^{k-2}\zeta_2(T) \\ S'(T, 0) &\simeq \gamma + \ln\{S(T, 1)\} - \tfrac{1}{2}\zeta_2(T)/\{S(T, 1)\}^2 \end{aligned}\right\} \quad . \quad (27)$$

where $\zeta_2(T) = 1 + 1/2^2 + \ldots 1/T^2$; and S' is the derivative of S with respect to k. Then, from equations (20) and (26), since $\xi_1' \equiv \xi_1$

$$\frac{\xi_1 - \xi_T'}{\sigma_1} = \frac{k\,!\,|S(T, k) - 1|}{\{(2k)\,! - (k\,!)^2\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad (28)$$

As $k \to 0$, $\xi_1 - \xi_T' \to 0{\cdot}78\,S'(T, 0)$

$$= 0{\cdot}78\left\{\frac{T(T-1)}{1.2}\ln 2 - \frac{T(T-1)(T-2)}{1.2.3}\ln 3 + \ldots\right\} \quad . \quad . \quad (29)$$

TABLE 2. VALUES OF THE RATIO $(\xi_1 - \xi_T')/\sigma_1$

σ_1/σ_1	T 10	25	50	100
0·85	0·87	0·99	1·06	1·11
0·90	1·01	1·16	1·25	1·33
0·95	1·12	1·31	1·42	1·53
1·00	1·22	1·45	1·59	1·71
1·05	1·31	1·57	1·74	1·88
1·10	1·38	1·69	1·89	2·05
1·15	1·45	1·80	2·01	2·22
1·20	1·51	1·90	2·12	2·37

$S'(T, 0)$ can be computed accurately from equation (27) for $T \geqslant 2$ and hence we can obtain the values of $(\xi_1 - \xi_T')/\sigma_1$ for $k = 0$.

$S(T, k)$ can be computed accurately for the range of k dealt with, from equation (27) for $T \geqslant 2$ and hence the value of $(\xi_1 - \xi_T')/\sigma_1$ for non-zero k. The values of the ratio are given in Table 2.

(*f*) *The average range among sets of T annual maxima*

The average range $= \xi_T - \xi_T' = (\xi_T - \xi_1) + (\xi_1 - \xi_T')$. Hence

$$\frac{\xi_T - \xi_T'}{\sigma_1} = \frac{k\,!\,|S(T, k) - T^{-k}|}{\{(2k)\,! - (k\,!)^2\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad (30)$$

The average range, expressed as the ratio $(\xi_T - \xi_T')/\sigma_1$, is given in Table 3. The average range in normal samples (Tippett 1925) is added.

TABLE 3. VALUES OF THE RATIO $(\xi_T - \xi_T')/\sigma_1$

σ_1/σ_2	T 10	25	50	100
0·85	2·72	3·89	4·93	6·12
0·90	2·86	3·96	4·86	5·83
0·95	2·96	3·97	4·75	5·55
1·00	3·01	3·95	4·64	5·30
1·05	3·04	3·90	4·54	5·09
1·10	3·05	3·87	4·46	4·94
1·15	3·06	3·86	4·38	4·84
1·20	3·07	3·85	4·32	4·77
Normal	3·08	3·96	4·50	5·02

(*g*) *Maximum values likely to be exceeded once in T years*

From equation (2), the value x is exceeded on $f(x) = \exp(-y)$ occasions in a year; that is, once in $T' = \exp y$ years.

But if we consider only frequencies of annual maximum values, then, from equation (3), the probability of a value x being exceeded is $\{1 - \exp(-e^{-y})\}$; that is, it is exceeded once in T years where

$$1/T = 1 - \exp(-e^{-y})$$

We shall deal here in detail only with the frequency of occurrence of annual maximum values.

The annual maximum value x which is likely to be exceeded once in T years is determined by $x = a(1 - e^{-ky})$ where $\exp(-e^{-y}) = 1 - (1/T)$.

Then $$x - \xi_1 = a(k\,! - e^{-ky})$$

$$\frac{x - \xi_1}{\sigma_1} = \frac{k\,! - \{\ln [T/(T-1)]\}^k}{\pm \{(2k)\,! - (k\,!)^2\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad (31)$$

where the denominator has the sign of k. The ratios $(x - \xi_1)/\sigma_1$, with corresponding values of y, are given in Table 4.

TABLE 4. VALUES OF $(x - \xi_1)/\sigma_1$

σ_1/σ_2	T	1·58	5	10	25	50	100	250
	y	0	1·50	2·25	3·20	3·90	4·61	5·52
0·85		− 0·44	0·48	1·05	1·95	2·76	3·72	5·22
0·90		− 0·45	0·58	1·18	2·06	2·78	3·60	4·79
0·95		− 0·46	0·66	1·26	2·07	2·71	3·38	4·31
1·00		− 0·45	0·72	1·31	2·04	2·59	3·14	3·85
1·05		− 0·44	0·75	1·34	2·00	2·47	2·91	3·45
1·10		− 0·42	0·77	1·34	1·93	2·32	2·67	3·09
1·15		− 0·39	0·79	1·34	1·85	2·19	2·47	2·78
1·20		− 0·36	0·81	1·33	1·79	2·08	2·30	2·52
Normal		− 0·34	0·84	1·27	1·75	2·06	2·33	2·66

3. NOTES ON PRACTICAL PROCEDURE

(a) *Further remarks on plotting the y, x curves*

The method of plotting used to obtain the y, x curves of Figs. 1 and 2 was to assign the 'compromise' frequency $(m - \frac{1}{2})/n$ (Hazen 1930) to the mth value in a series of n ordered maxima. This is convenient in practice; but it should be remembered that any curve obtained from this kind of plotting is only an estimate of the theoretical curve. The theoretical curve can be drawn, after the values of ξ_1, σ_1 and σ_2 have been computed, by plotting the values of x obtained from Table 4 for

$T =$	1·58	5	10	25	50	100	250
$y =$	0	1·50	2·25	3·20	3·90	4·61	5·52

Two examples, each showing plotting according to the 'compromise' probability $(m - \frac{1}{2})/n$ together with the theoretical curve, are given in Fig. 3 (a) and (b). The examples are of two kinds of hydrological data. The curve (a) is of Type I and (b) of Type III.

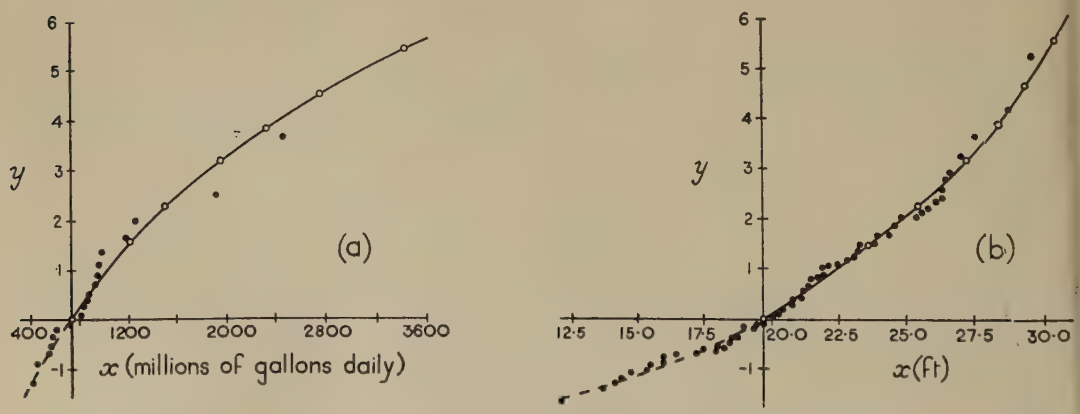

Figure 3. y, x graphs of distribution of maximum values, using the 'compromise' frequency $(m - \frac{1}{2})/n$, together with the theoretical curves. (a) Annual maximum discharge, Little River Catchment Area, Westfield, Mass., 1910-1928; (b) annual maximum flood stage, Hartford, Conn., 1843-1934.

Authorities for data: Hazen (1930); Barrows, H. K. 1948 *Floods*.

Annual maximum values which are exceeded once in T years can be determined since

$$y = -\ln\ln\{T/(T-1)\} \quad . \quad . \quad . \quad (32)$$

and the corresponding value of x can be read from the theoretical curve.

Values x (whether maximum or not) which are exceeded once in T' years can also be determined graphically from the theoretical curve. We have

$$y = \ln T' \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and the corresponding value of x can be read off.

(*b*) *An example of the plotting and computations for the Hartford data*

Table 5 gives corresponding values of P, y and x for the first five and last five of the 92 annual maximum floor stages arranged in order of magnitude.

TABLE 5. CORRESPONDING VALUES OF P, y AND x FOR DATA OF ANNUAL MAXIMUM FLOOD STAGES (FT.) AT HARTFORD, 1843-1934. $n = 92$.

m	$P=(2m-1)/2n$	y	$x = X_m$
1	0·005	−1·67	12·2
2	0·016	−1·43	13·8
3	0·027	−1·28	14·2
4	0·038	−1·19	14·5
5	0·049	−1·11	14·8
88	0·951	2·99	26·7
89	0·962	3·26	27·2
90	0·973	3·61	27·7
91	0·984	4·18	29·0
92	0·995	5·21	29·8

TABLE 6. ANNUAL MAXIMUM FLOOD STAGE (FT) AT HARTFORD, 1843-1934. COMPUTATION OF PARAMETERS.

X_m	m	f	f'	x	fx	fx^2	$f'x$	$f'x^2$
12	1	1	1	−9	−9	9	−9	81
14	2, 3	2	8	−7	−14	98	−56	392
15	4-7	4	40	−6	−24	144	−240	1,440
16	8-11	4	72	−5	−20	100	−360	1,800
17	12-14	3	75	−4	−12	48	−300	1,200
18	15-18	4	128	−3	−12	36	−384	1,152
19	19-29	11	517	−2	−22	44	−1,034	2,068
20	30-38	9	603	−1	−9	9	−603	603
21	39-59	21	2,037	0				
22	60-65	6	744	1	6	6	744	744
23	66-73	8	1,104	2	16	32	2,208	4,416
24	74-76	3	447	3	9	27	1,341	4,023
25	77-81	5	785	4	20	80	3,140	12,560
26	82-86	5	835	5	25	125	4,175	20,875
27	87-89	3	525	6	18	108	3,150	18,900
28	90	1	179	7	7	49	1,253	8,771
29	91	1	181	8	8	64	1,448	11,584
30	92	1	183	9	9	81	1,647	14,823
Totals		92	8,464		−4	1,060	16,120	105,432

$\xi_1 = -0{\cdot}04 = 21{\cdot}0$ ft
$\sigma_1^2 = 11{\cdot}52 - 0{\cdot}02 = 11{\cdot}50$
$\sigma_1 = 3{\cdot}39$ ft

$\xi_2 = 1{\cdot}90$
$\sigma_2^2 = 12{\cdot}46 - 3{\cdot}61 = 8{\cdot}85$
$\sigma_2 = 2{\cdot}97$ ft

$\sigma_1/\sigma_2 = 1{\cdot}14$

The corresponding values of y and x are plotted in Fig. 3 (b) as dots.

Table 6 shows the computation of the basic parameters for Hartford. σ_1 is computed from the set $X_1, X_2 \ldots X_m \ldots X_n$ in the usual way. σ_2 is computed from the same set with the weights 1, 3, $\ldots (2m-1) \ldots (2n-1)$ respectively assigned to them. (The reasons for this are given in Appendix 2.) The respective frequencies are f and $f' = \Sigma(2m-1)$; and x is referred to an arbitrary origin at 21·0 ft.

By interpolating for the value of $\sigma_1/\sigma_2 = 1{\cdot}14$ in Table 4 we obtain the values of the annual maximum value x to be expected once in T years. They are given in Table 7.

TABLE 7. ANNUAL MAXIMUM FLOOD STAGE x (ft) AT HARTFORD, EXPECTED ONCE IN T YEARS.

T	1·58	5	10	25	50	100	250
y	0	1·50	2·25	3·20	3·90	4·61	5·52
$\frac{x-\xi_1}{\sigma_1} = \frac{x-21{\cdot}0}{3{\cdot}39}$	− 0·39	0·79	1·34	1·87	2·22	2·51	2·84
x	19·7	23·7	25·5	27·3	28·5	29·5	30·6

The corresponding values of y and x are plotted in Fig. 3 (b) as circles. The curve drawn through them is the theoretical curve $x = a(1 - e^{-ky})$.

The annual maximum value likely to be exceeded say once in 3 years can be obtained graphically from the theoretical curve, using expression (32)

$$y = -\ln\ln\{T/(T-1)\}$$
$$= -\ln\ln 1{\cdot}5 = 0{\cdot}90$$

The corresponding value of x from Fig. 3 (b) is 22·3 ft.

The value, whether an annual maximum or not, likely to be exceeded once in 3 years can be obtained by using expression (33)

$$y = \ln T' = \ln 3 = 1{\cdot}10$$

The corresponding value of x, from Fig. 3 (b), is 22·8 ft.

The difference between the two probability concepts may be considerable for small T; but for large T there is little difference. It is advisable to keep the distinction: we may be interested in all occurrences beyond a certain value, or only in the largest of such values in any one year.

(c) *Other examples of the use of Tables 1, 2 and 4*

Table 8 gives for Westfield the annual maximum flood discharge to be expected once in T years (use of Table 4).

TABLE 8. ANNUAL MAXIMUM FLOOD DISCHARGE, x (10^6 GALLONS DAILY) EXPECTED ONCE IN T YEARS AT WESTFIELD. $\xi_1 = 943$, $\sigma_1 = 494$, $\sigma_1/\sigma_2 = 0{\cdot}88$

T	1·58	5	10	25	50	100	250
y	0	1·50	2·25	3·20	3·90	4·61	5·52
$\frac{x-\xi_1}{\sigma_1} = \frac{x-943}{494}$	− 0·45	0·54	1·13	2·02	2·78	3·65	4·96
x	721	1,210	1,502	1,941	2,316	2,746	3,493

As an example of the use of Tables 1 and 2, computed values of ξ_{10}, ξ_{25}, ξ_{50}, ξ_{100}; ξ_{10}', ξ_{25}', ξ_{50}', ξ_{100}', together with recorded extremes, are given in Table 9 for the 12 examples shown in Figs. 1 to 3.

TABLE 9. COMPUTED VALUES OF ξ_{10}, ξ_{25}, ξ_{50}, ξ_{100}; ξ_{10}', ξ_{25}', ξ_{50}', ξ_{100}'.

Fig.	Example	Units	ξ_1	σ_1	σ_1/σ_2	No. of years
1 (a)	Naples; maximum hourly rainfall	mm	27	14	0·93	46
1 (b)	Marseilles; maximum daily rainfall	mm	67	35	0·90	65
1 (c)	Tripoli; maximum four-day rainfall	mm	81	37	0·92	30
1 (d)	Milan; maximum vapour pressure	mb	18·9	1·1	1·00	30
1 (e)	Siros; maximum August temperature	°C	35·0	2·3	1·18	32
1 (f)	Marseilles; maximum pressure	mb	1,034	3·5	1·09	24
1 (g)	Trieste; maximum hourly mean wind	km/hr	108	6·7	1·14	18
1 (h)	Tateno; maximum winter 9-km wind	m/sec	101	19	1·11	3
2 (a)	Athens; minimum temperature	°C	− 1·5	1·9	1·14	30
2 (b)	Tortosa; minimum pressure	mb	989	5·2	1·03	25
3 (a)	Westfield; maximum flood discharge	10^6 gal/day	943	494	0·88	19
3 (b)	Hartford; maximum flood stage	ft	21·0	3·4	1·14	92

TABLE 9 (continued).

Fig.	ξ_{10}	ξ_{25}	ξ_{50}	ξ_{100}	Recorded extreme	ξ_{10}'	ξ_{25}'	ξ_{50}'	ξ_{100}'	Recorded extreme
1 (a)	53	65	75	86	73	12	10	8	7	10
1 (b)	132	165	193	225	221	32	27	23	20	25
1 (c)	149	182	205	237	207	42	36	32	29	31
1 (d)	20·9	21·6	22·3	22·8	21·5	17·6	17·3	17·1	17·0	17·2
1 (e)	38·6	39·5	40·2	40·7	40·0	31·6	30·7	30·2	29·7	30·8
1 (f)	1,040	1,042	1,043	1,045	1,042	1,029	1,028	1,027	1,027	1,028
1 (g)	119	122	124	126	121	98	96	95	93	96
1 (h)	144	154	162	170	136	75	68	65	61	75
2 (a)	− 4·6	− 5·5	− 6·1	− 6·6	− 5·5	1·3	1·9	2·3	2·7	1·8
2 (b)	980	977	974	972	976	996	997	998	998	996
3 (a)	1,857	2,346	2,676	3,285	2,452	474	405	366	332	414
3 (b)	26·5	28·1	29·2	30·3	29·8	16·1	15·0	14·2	13·5	12·2

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Appendix 1. Corresponding values of P and $y = -\ln\ln(1/P)$.

P	y	P	y	P	y	P	y	P	y
0·01	− 1·53	0·21	− 0·44	0·41	0·11	0·61	0·70	0·81	1·56
0·02	− 1·36	0·22	− 0·41	0·42	0·14	0·62	0·74	0·82	1·62
0·03	− 1·25	0·23	− 0·39	0·43	0·17	0·63	0·77	0·83	1·68
0·04	− 1·17	0·24	− 0·36	0·44	0·20	0·64	0·81	0·84	1·75
0·05	− 1·10	0·25	− 0·33	0·45	0·23	0·65	0·84	0·85	1·82
0·06	− 1·03	0·26	− 0·30	0·46	0·25	0·66	0·88	0·86	1·89
0·07	− 0·98	0·27	− 0·27	0·47	0·28	0·67	0·91	0·87	1·97
0·08	− 0·93	0·28	− 0·24	0·48	0·31	0·68	0·95	0·88	2·06
0·09	− 0·88	0·29	− 0·21	0·49	0·34	0·69	0·99	0·89	2·15
0·10	− 0·83	0·30	− 0·19	0·50	0·37	0·70	1·03	0·90	2·25
0·11	− 0·79	0·31	− 0·16	0·51	0·40	0·71	1·07	0·91	2·36
0·12	− 0·75	0·32	− 0·13	0·52	0·42	0·72	1·11	0·92	2·48
0·13	− 0·71	0·33	− 0·10	0·53	0·45	0·73	1·16	0·93	2·62
0·14	− 0·67	0·34	− 0·08	0·54	0·48	0·74	1·20	0·94	2·78
0·15	− 0·64	0·35	− 0·05	0·55	0·51	0·75	1·25	0·95	2·97
0·16	− 0·61	0·36	− 0·02	0·56	0·55	0·76	1·29	0·96	3·20
0·17	− 0·57	0·37	0·01	0·57	0·58	0·77	1·34	0·97	3·49
0·18	− 0·54	0·38	0·03	0·58	0·61	0·78	1·39	0·98	3·90
0·19	− 0·51	0·39	0·06	0·59	0·64	0·79	1·45	0·99	4·61
0·20	− 0·47	0·40	0·09	0·60	0·67	0·80	1·50		

Appendix 2. Representation of the set of two-year maxima

The value of σ_1 is obtained from a given set of X's, representing the maxima (or minima) for a period of say n years.

To obtain a value of σ_2 which is consistent with and comparable with this value of σ_1 the whole set $X_1, X_2, \ldots X_n$ must be used.

If we consider the set $X_1, X_2, \ldots X_n$ to be a fair sample of the one-year maxima, then we may also take a population with r each of these members as a fair sample of the one-year maxima.

The r members X_m may be considered as differing from each other by infinitesimally small amounts. Then in selecting pairs of values from the rn values, the r members X_m will be the greater member of a pair on $(m-1)\,r, (m-1)\,r+1, \ldots (rm-1)$ occasions respectively; that is a total of $\frac{1}{2}\,r\{r\,(2m-1)-1\}$ occasions.

Now as $r \to \infty$ we see that the population of two-year maxima is represented by the X_m with weights $2m-1$.

Appendix 3. Solution of the Fisher-Tippett functional equation

The equation has been written (13) as

$$P^s(x) = P(a_s\,x + b_s)$$

where a_s and b_s are functions of s. In our notation $P = \exp(-e^{-y})$. Substitute for P in equation (13) and take logarithms twice, whence

$$\ln s - y(x) = -y(a_s\,x + b_s) \qquad . \qquad . \qquad . \qquad (34)$$

Expand $y(x)$ and $y(a_s\,x + b_s)$ in powers of $(x - x_0)$, where $y(x_0) = 0$. Then equation (34) may be written

$$\ln s = -y(b_s + a_s\,x_0) + (x - x_0)\{y'(x_0) - a_s\,y'(b_s + a_s\,x_0)\} + \frac{(x-x_0)^2}{2!}\{y''(x_0) - a_s^2\,y''(b_s + a_s\,x_0)\} + \ldots \qquad . \qquad . \qquad (35)$$

Since equation (35) holds for the range of values of x, we derive the set of equations

$$\left.\begin{aligned} -y(b_s + a_s x_0) &= \ln s \\ y'(x_0) - a_s y'(b_s + a_s x_0) &= 0 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ y^{(n)}(x_0) - a_s^n y^{(n)}(b_s + a_s x_0) &= 0 \end{aligned}\right\} \quad . \quad . \quad (36)$$

where the bracketed index denotes the nth derivative. Then, from equation (36) we have, for $n \geqslant 1$

$$\frac{y^{(n)}(b_s + a_s x_0)}{\{y'(b_s + a_s x_0)\}^n} = \frac{y^{(n)}(x_0)}{\{y'(x_0)\}^n} \quad . \quad . \quad . \quad (37)$$

The right-hand side of equation (37) is a function of n only, and may be written as c_n. Since, for varying values of s, $(b_s + a_s x_0)$ takes all values in the range of x, we have generally for $n \geqslant 1$

$$\frac{y^{(n)}(x)}{\{y'(x)\}^n} = c_n$$

or

$$y^{(n)}(x) = c_n \{y'(x)\}^n \quad . \quad . \quad . \quad . \quad (38)$$

Differentiating equation (38) with respect to x we have

$$\begin{aligned} y^{(n+1)}(x) &= n \,.\, c_n \{y'(x)\}^{n-1} \,.\, y''(x) \\ &= n \,.\, c_n \{y'(x)\}^{n-1} \,.\, c_2 \{y'(x)\}^2 \quad \text{from equation (38)} \\ &= n \,.\, c_2 c_n \{y'(x)\}^{n+1} \quad . \quad . \quad . \quad . \quad . \quad (39) \end{aligned}$$

Hence we derive the recurrence relation for c_n

$$c_{n+1} = n \,.\, c_2 c_n \quad \text{for} \quad n \geqslant 1 \quad . \quad . \quad . \quad . \quad (40)$$

whence

$$c_n = (n-1)!\, c_2^{n-1} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Hence from equation (38)

$$y^{(n)}(x_0) = (n-1)!\, c_2^{n-1} \{y'(x_0)\}^n \quad . \quad . \quad . \quad (42)$$

The expansion for $y(x)$ in powers of $(x - x_0)$ is

$$y(x) = \sum_{n=1}^{\infty} \frac{(x - x_0)^n}{n!} y^{(n)}(x_0)$$

which, on substituting for $y^{(n)}(x_0)$ from equation (42), and writing w for $y'(x_0)$, k for c_2 and a for $1/kw$, gives

$$y(x) = \frac{1}{k} \sum_{n=1}^{\infty} \frac{(x - x_0)^n}{n a^n} \quad . \quad . \quad . \quad . \quad (43)$$

that is

$$\left.\begin{aligned} y(x) &= -\frac{1}{k} \ln\left(1 - \frac{x - x_0}{a}\right) \\ (x - x_0) &= a(1 - e^{-ky}) \\ y'(x) &= 1/k \{a - (x - x_0)\} \end{aligned}\right\} \quad . \quad . \quad (44)$$

From the first two equations of (36), after some simplification, we find that

$$\begin{aligned} a_s &= s^k \\ b_s &= (x_0 + a)(1 - s^k) \end{aligned} \quad . \quad . \quad . \quad . \quad (45)$$

so that equation (13) can be written

$$P^s\{(x_0 + (x - x_0)\} = P\{x_0 + a(1 - s^k) + s^k (x - x_0)\} \quad . \quad . \quad (46)$$

We may arbitrarily assign the origin of x to where $y = 0$, when equation (44) gives the x, y relationship in the form of equation (8)

$$x = a(1 - e^{-ky}).$$

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Two-way relationship between the snow or ice limit and 1,000-500 mb thicknesses in the overlying atmosphere

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SUMMARY

Statistical studies of 1,000-500 mb thickness, (i) occurring with frozen or unfrozen forms of precipitation and (ii) prevailing at various distances from the snow or ice limit over wide regions with snow or ice cover, over bare ground and over open sea are used to discover (*a*) the likelihood that precipitation will take the form of rain or snow in given circumstances and (*b*) the behaviour of the thickness when well-established and extensive snow or ice fields are encountered. The results indicate an important influence produced by snow fields of the order of 1,000 mi and more in longitudinal extent upon the general circulation of the atmosphere.

Choice of 1,000-500 mb thickness as an indicator rests upon its adequacy and upon its general use and availability in forecasting offices; some preliminary argument is devoted to showing that the choice is reasonable on these grounds and that the lack of refinement involved in considering thicknesses of so deep a layer is not serious.

1. INTRODUCTION

1,000-500 mb thickness maps (relative topography) and the associated 500-mb contours (absolute topography of the 500-mb surface) have become the main tools of the busy forecaster concerned with co-ordination of many upper-air data, and of the research analyst studying broad-scale weather patterns. The 500-mb level is the highest for which enough reliable data are available for analysis over wide areas, and these charts provide a good synopsis for study of the basic thermal and dynamical problems.

The investigation here described was undertaken firstly to establish a readily usable criterion for the forecaster with regard to thickness values and the likelihood of rain or snow falling, and secondly to probe in more detail an evident association between the 17,400-ft thickness isopleth and the contemporary limit of snow-cover and of close pack ice on the polar seas.

The results suggest that a widespread snow-cover behaves as an important control reacting upon the atmospheric circulation itself.

2. VERTICAL TEMPERATURE DISTRIBUTION AND 1,000-500 MB THICKNESS

To be able to follow the working of climatic controls we commonly focus our attention on particular thickness isopleths, conveniently marked by standard colours on the working charts, as introduced by Sutcliffe. Details are given in Table 1.

TABLE 1. STANDARD THICKNESS ISOPLETHS AND CORRESPONDING TEMPERATURES

1,000-500 mb thickness (ft)	1,000-500 mb thickness (m)	Corresponding temperature at 1,000 mb (for saturated adiabatic lapse rate throughout) (°C)	(°F)	Standard colour of isopleths
19,200	5,850	27	81	purple
18,600	5,670	20	68	red
18,000	5,490	13	55	green
17,400	5,310	6	43	blue
16,800	5,130	− 1·5	29	brown
16,200	4,940	− 9	16	purple
15,600	4,760	− 16·5	2	red

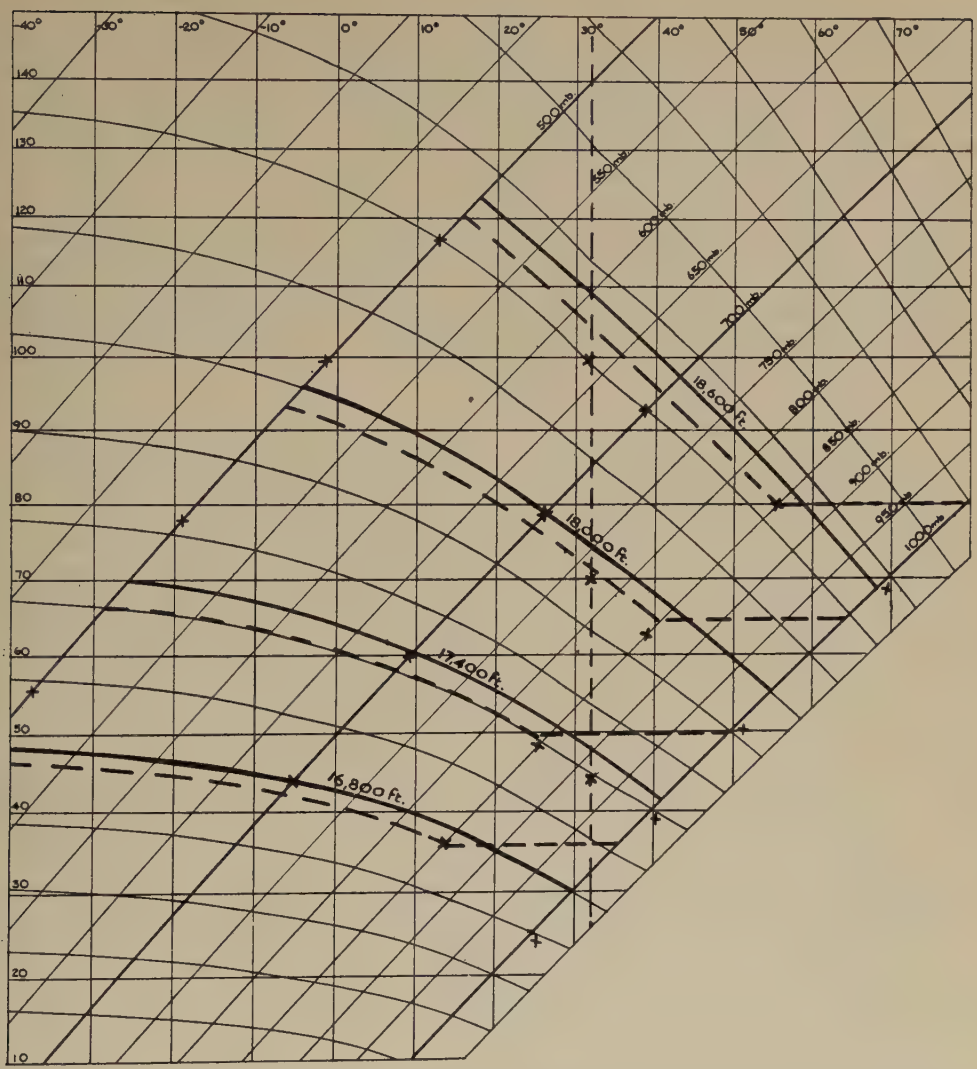

Figure 1. The distribution of temperatures aloft corresponding to particular thickness values in certain circumstances : (i) *Full lines* – saturated adiabatic lapse rate throughout the 1,000-500 mb layer. (ii) *Pecked lines* – dry adiabatic lapse rate from 1,000 mb up to the 850-mb level, saturated adiabatic lapse rate from 850 to 500 mb (**x** on diagram marks mean of observed temperature (Table 2); surface values plotted at 1010 mb).

It will be seen by eye that strong inversions of temperature confined to the surface layer (often with pressures over 1,000 mb) must have little effect on the temperatures above 900 or 950 mb corresponding to particular thickness values. Larger positive temperature departures in the upper levels from the curves shown in Fig. 1 are only likely to occur with deep inversions or isothermal layers, i.e., with a few fronts and occasionally in other situations in continental interior or inner arctic regions.

To the experienced aviation forecaster, who is often acutely aware of all the variations of temperature lapse in the lower atmosphere, which may be covered by a single thickness value, and of all the finer structure of frontal layers and subsidence surfaces which may be masked by a smooth ' total thickness ' pattern, the very limited range of temperatures aloft commonly occurring with a given thickness value may come as a surprise.

In warm climates, where the air may have a high moisture content, temperatures indicated by the curves of state in Fig. 1 should be virtual temperatures, exceeding the dry-bulb temperature by up to 5°C (9°F) – the figure for saturated air at an observed temperature of 30°C (86°F). At normal surface temperatures in middle latitudes the correction for virtual temperature is seldom more than 1 to 2°C, and, at the freezing point or below, the discrepancy is quite negligible.

In order to form a more realistic gauge of the dry-bulb temperatures actually occurring with the particular 1,000-500 mb thickness corresponding to our standard lines, all available upper-air ascents from stations in western, central and northern Europe and from the weather ships in the eastern north Atlantic for the year 1951 which gave thickness values of just 16,800; 17,400; 18,000 or 18,600 within ± 30 ft were statistically examined.* The results of this examination are set forth in Table 2.

TABLE 2. OBSERVED TEMPERATURES WITH PARTICULAR 1,000-500 MB THICKNESS VALUES
(M = mean, E = extremes)

1,000-500 mb thickness (ft)	Temperatures (°F) Surface M	SD	E	850 mb M	SD	E	700 mb M	SD	E	500 mb M	SD	E	No. of cases
16,800	25·7	10·3	37	11·3	3·8	17	− 5·5	2·4	− 2	− 36·9	5·4	− 27	18
			− 1			5			− 9			− 44	
17,400	40·1	5·6	52	25·5	2·9	33	9·6	2·3	16	− 18·8	4·2	− 10	160
			28			18			3			− 28	
18,000	51·6	6·2	63	38·7	2·9	44	27·0	2·1	31	− 1·1	2·4	4	41
			33			32			21			− 6	(July, Nov., Dec. only)
18,600	69·7	7·6	84	55·5	3·1	64	39·7	2·3	45	12·8	3·2	23	74
			50			49			34			5	

The standard deviations in the case of the smaller samples in Table 2, and elsewhere in this paper, were based on the formula

$$\left\{\frac{\Sigma (x - \bar{x})^2}{n - 1}\right\}^{\frac{1}{2}}$$

To reduce the work to manageable proportions, the 18,000-ft thickness cases studied were limited to those occurring in July, November and December.

Checking weeded out some doubtful ascents. In the end, 293 ascents from these 39 places were used and are summarized in Table 2.

The striking feature of Table 2 is that standard deviations are only about 3°F at all the upper levels, save for the top of the 1,000-500 mb layer in the coldest ascents. The actual mean temperatures are close to the standard curves in Fig. 1.

Many different groupings of the ascents according to time of day, time of year and geographical position were tried. The resulting mean temperatures differed in nearly all cases by less than 2°F at 500 and 700 mb and, in most cases, also at 850 mb from the overall means in Table 2. Greater differences occurred at the ground for obvious reasons (ground inversions, diurnal variation, heating over warm water, etc.). Only the groups of cases picked out in Table 3 were of any special interest.

* The stations examined were : British ocean weather ships near 59°N 19°W and 52°N 20°W; Thorshavn, Lerwick, Stornoway, Leuchars, Aldergrove, Valentia, Liverpool, Larkhill, Camborne, Downham Market, Sola, Gardermoen, Torslanda, Stockholm, Riga, Warsaw, Copenhagen (Kastrup), Flensburg, Jever, Iserlohn, Hanover, Berlin (Tempelhof), Greifswald, Wernigerode, Dresden, Erlangen, Munich (surface temperatures not used, altitude 1,700 ft), Wiesbaden, Utrecht (De Bilt), Brussels (Uccle), Paris (Trappes), Strasbourg, Bordeaux, Lyons, Nîmes, Payerne, (surface temperatures not used, altitude 1,500 ft), Vienna.

TABLE 3. OBSERVED TEMPERATURES IN SELECTED SITUATIONS

1,000-500 mb thickness (ft)	Mean temperature °(F) Ground	850 mb	700 mb	500 mb	No. of cases	Basis of selection	
16,800	**18·3**	9·1	— 4·7	— 33·4	9	Continental group	Stockholm Torslanda Oslo
16,800	31·0	13·4	— 6·4	— 40·4	9	Oceanic group	Ships Thorshavn Lerwick Stornoway
17,400	**44·2**	25·0	**8·6**	— 17·6	22	Weather ships	(all cases)
18,600	**62·0**	55·0	**41·9**	**15·5**	8	Oceanic group	Ships Valentia Camborne

Comparison of the cases in Table 3 with Table 2 brings out chiefly the great difference of behaviour of surface temperature between oceanic and inland stations, including those in the British Isles. The figures in bold type in Table 3 appear to differ from their counterparts in Table 2 by amounts that have only a 1 in 20 probability (or less) of occurring by chance (*t* tests applied), even in small samples of the size here used.

The greatest standard deviation was of surface temperatures observed at inland stations with 16,800-ft thickness, namely **11·2**°F, whereas the S.D. of surface temperature at oceanic stations was under **3**°F alike with thicknesses of 16,800 and 18,600 ft (this variance ratio for 16,800 ft cases has a probability of less than 1 in 1,000 of occurring by chance – *F* test applied).

Table 4 presents a survey of the behaviour of the freezing level in all the cases covered by Table 2.

Summarizing, we find that the thickness value alone will give us the temperature at any given height in the free air to within 3°F, and the freezing level to within about a thousand feet, in a considerable majority of cases over the European and Atlantic region.

TABLE 4. OBSERVED FREEZING LEVELS WITH PARTICULAR 1,000-500 MB THICKNESS VALUES

1,000-500 mb thickness (ft)	Average level of (lowest) 0°C isotherm (mb)	S.D. (mb)	No. of cases	Lowest observed 0°C isotherm (mb)	Highest observed 0°C isotherm (mb)	Percentage of all cases examined which showed double or triple freezing levels
16,800	Very near surface	–	18	Surface	960	0
17,400	915	37	160	Surface	850	4·4
18,000	770	34	41	860	710	12·2
18,600	635	21	74	670	570	2·7

3. THE FREEZING-LEVEL CRITERION FOR RAIN OR SNOW

The height of the freezing level above the ground and the thermal structure of the lowest few thousand feet of the atmosphere largely determine whether snow can reach the ground. A brief survey of this point is essential before proceeding further.

Evaporation (sublimation) and melting are the processes whereby falling snowflakes, in all their variety of forms and sizes, may be prevented from reaching the ground. The

considerable air resistance of the larger flakes, and any rising air currents encountered, give more time for saturation-deficit and temperature-excess in the surrounding air to dispose of the snow. Terminal velocities of small snowflakes of about 5 mm diameter are only about 1·1 m/sec (Giblett 1921) against 8·0 m/sec, for raindrops or hailstones of the same cross-section; moreover the quantity of ice to be melted in the snowflake is only of the order of one-tenth of that in the hailstone. Once a snowflake falls into air which is above the freezing point, it is probably the time-integral of the temperature-difference, air *minus* snowflake, which is most effective in preventing snow reaching the ground. Large flakes fall a little more quickly; the terminal velocity is about 1·8 m/sec for flakes 20 mm across (Giblett 1921). Evaporation cooling, the greater latent heat required and the shorter duration of fall often suffice to preserve big snowflakes until they reach the ground after falling through air warm enough to dispose of smaller flakes and grains of snow entirely. It must be remembered, however, that falling snow and sleet, and the latent heats of evaporation and fusion abstracted, gradually chill air which may have been initially well above the freezing point.

Murray (1952) examined the occurrence of rain (and rain showers) or snow (and snow showers) with various freezing-level heights in the British Isles and found that frozen and unfrozen forms became equally likely when the freezing level is at about 1,000 ft. This figure seems quite generally applicable to continental and oceanic locations.

Hail in its larger forms is a different problem, which for the purpose of the present study we shall ignore (see, however, later Section 4 (*c*)).

The snow problem is complicated somewhat for the forecaster by time-variations of freezing level within one air mass. Quick, erratic variations of several hundred feet occur in instability showers: these are usually fleeting, but under favourable circumstances a new snow carpet may result from showers starting with a general freezing level above 1,000 ft. Douglas (1950) noticed that long-continued rain and sleet falling through dry air may so cool the atmosphere that the freezing level drops by as much as 4,000 to 5,000 ft (forming a deep isothermal layer at about 0°C) and snow ultimately reaches the ground. Widespread snowfalls in southern England began in this way on 25 December 1927 and 1 November 1942; the same effect is both frequent and important in central Europe where low relative humidities, favouring evaporation cooling, are much commoner than in England.

These considerations come into play with 1,000-500 mb thicknesses from 17,100 ft up to 17,500 or, at most, 17,600 ft in connexion with snowfall over regions of general elevation less than 100 to 150 m (300 to 500 ft).

4. The thickness criterion for snowfall : precipitation in frozen and unfrozen forms

Murray (1952) statistically examined the occurrence of precipitation in frozen and unfrozen forms with various thicknesses of the 1,000-500 and 1,000-700 mb layers at or near seven British upper-air stations (Lerwick, Stornoway, Aldergrove, Leuchars, Liverpool, Downham Market and Camborne). Grouping these stations together inevitably masked the differentiation we have noticed between inland or continental and oceanic localities, most of his chosen stations being classable as oceanic. He found that rain and snow were equally likely when the 1,000-500 mb thickness was about 17,140 ft or the 1,000-700 mb thickness about 9,120 ft; these figures prove to be fairly typical of oceanic (Gulf Stream water) situations. Evidence of generally rather higher thickness values for the changeover from rain to snow may be detected in a recent paper on cold pools by Sumner (1953), who found a ' fairly distinct separation ' when the *minimum*

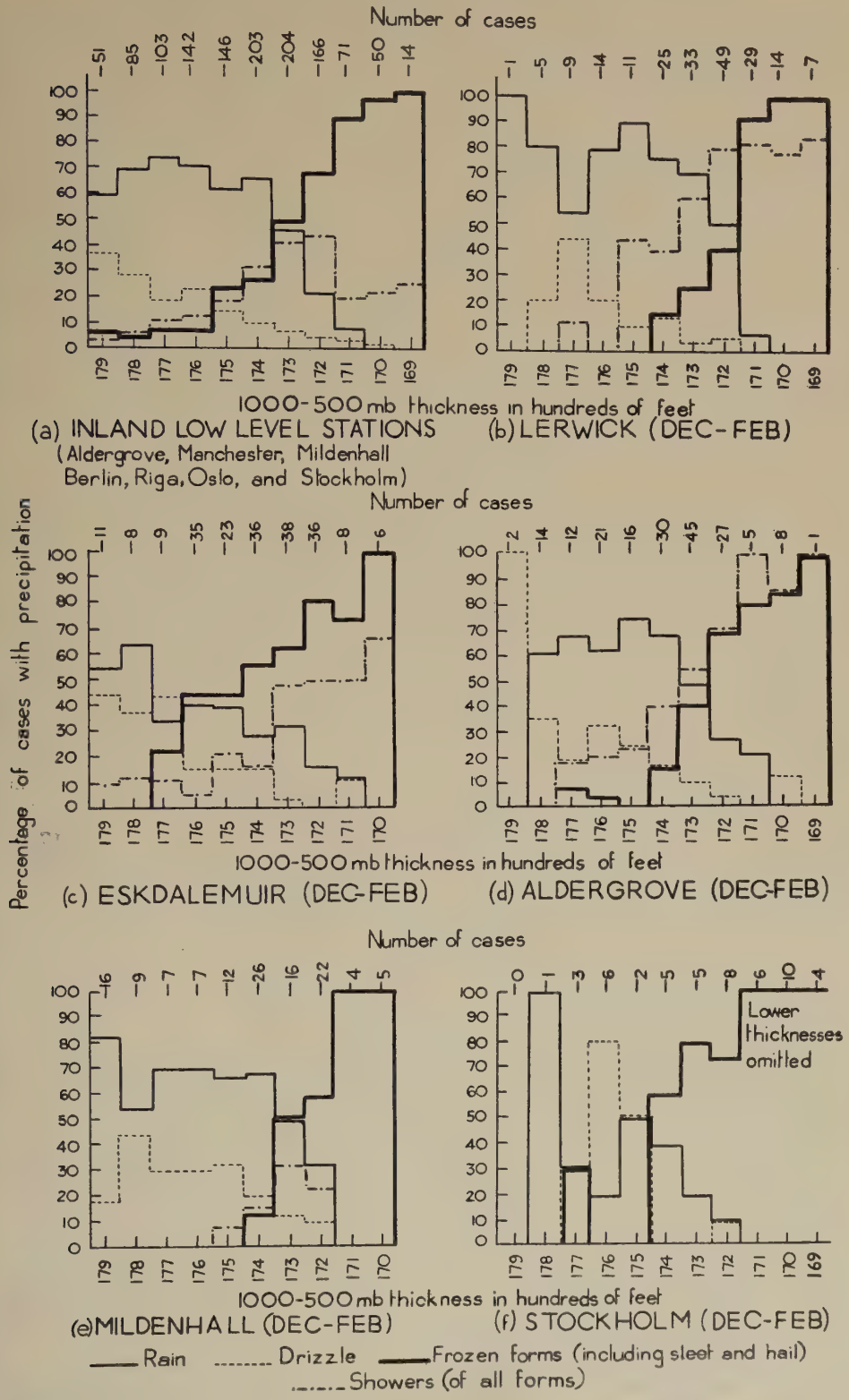
Number of cases
-51 -85 -103 -142 -146 -203 -204 -166 -71 -50 -14
-1 -5 -9 -14 -11 -25 -33 -49 -29 -14 -7
100 90 80 70 60 50 40 30 20 10 0
179 178 177 176 175 174 173 172 171 170 169
1000-500 mb thickness in hundreds of feet
(a) INLAND LOW LEVEL STATIONS
(Aldergrove, Manchester, Mildenhall
Berlin, Riga, Oslo, and Stockholm)
(b) LERWICK (DEC-FEB)
Number of cases
Percentage of cases with precipitation
-11 -8 -9 -35 -23 -36 -38 -36 -8 -6
-2 -14 -12 -21 -16 -30 -45 -27 -5 -8 -1
1000-500 mb thickness in hundreds of feet
(c) ESKDALEMUIR (DEC-FEB)
(d) ALDERGROVE (DEC-FEB)
Number of cases
-16 -9 -7 -7 -12 -26 -16 -22 -4 -5
-0 -1 -3 -6 -2 -5 -5 -8 -6 -10 -4
Lower thicknesses omitted
1000-500 mb thickness in hundreds of feet
(e) MILDENHALL (DEC-FEB)
(f) STOCKHOLM (DEC-FEB)
Rain
Drizzle
Frozen forms (including sleet and hail)
Showers (of all forms)

Figure 2. Frequencies of different forms of precipitation with different thickness values.

thickness in the cold pool was about 17,200 ft; we may safely assume that average thickness at the point where the precipitation was observed was somewhat higher (perhaps appreciably higher) than at the centre of the cold pool.

To establish the most general possible criterion, based upon a broader survey, the present investigator tabulated all cases of precipitation reported during the winter 1950-51 at the time (including precipitation in the last hour) and in the neighbourhood of the radio-soundings at Lerwick, Aldergrove, Liverpool (surface observations from Manchester, Ringway), Downham Market (Mildenhall), Berlin (Tempelhof), Riga, Stockholm and Oslo (Fornebu). These are all low-level stations, less than 300 ft above mean sea level; their mean latitude is 56°N; they comprise one oceanic location (Lerwick), various inland districts of the British Isles and several typical European continental locations. The months examined were November 1950 to April 1951 inclusive for the British stations and October 1950 to April 1951 inclusive for the others. In addition the precipitation forms at one higher-level station in the British Isles, Eskdalemuir (794 ft A.S.L.) were studied in relation to the contemporary 1,000-500 mb thickness values over the station taken from the published *Daily Aerological Record*.

In all there were 2,045 observations with precipitation to be classified.

The separate frequencies of rain, drizzle, snow, sleet, hail and showers of each type, also the total frequencies of precipitation of showery character, occurring with thickness ranges 17,960-18,050, 17,860-17,950 and so on down to 16,860-16,950 ft were examined for each of the nine stations named and in each month of the winter. Some of the results are shown in the histograms reproduced in Figs. 2 (*a*)-(*f*) and 3, in which sleet and hail are counted in with the frozen forms and shower frequencies include *all* forms of showers.

χ^2 tests applied to the changes of relative frequency of snow with different thicknesses represented in Figs. 2 (*a*)-(*f*) showed significant changes over the critical (changeover) ranges as follows : Fig. 2 (*a*) inland low-level group 17,100-17,400 ft (probability due to chance under 1 per cent), Fig. 2 (*b*) Lerwick 17,100-17,200 ft (chance probability under 1 per cent), Eskdalemuir 17,400-17,500 ft (chance probability under 1 per cent), Aldergrove 17,200-17,400 ft (chance probability 1 to 4 per cent), Mildenhall 17,300-17,400 ft (chance probability 3 per cent); Stockholm provided too few data for statistical verification.

Results for the whole collection may be summarized as follows :

(*a*) *Rain or frozen forms*

In general the changeover was commonest with a thickness of about 17,300 ft, at which figure frozen and unfrozen forms were equally frequent.

Over the warm waters of the North Atlantic Drift as at Shetland (water surface temperatures in winter about 10°C (50°F)) and the coastal places in the British Isles studied by Murray, the changeover occurs with a thickness of about 17,150 ft. In such situations snow only falls in air which is unstable (or nearly so) from the surface up.

Over already extensively snow-covered terrain, as at Oslo, Stockholm and Riga in the depth of winter, snow is as likely to fall as rain with thicknesses of 17,400 to 17,450 ft or even higher. Fig. 3 shows the histograms for frequency of different types of precipitation with different thickness values obtained by selecting all occasions when snow was lying (Oslo, Stockholm, Riga and Berlin). χ^2 tests on these data verified the changes of relative frequency of snow falling as significant (chance probability 1 per cent) over the critical range 17,200-17,600 ft.

The critical thickness at inland stations in the British Isles and at Berlin approximated *either* to values typical for Oslo-Stockholm-Riga or to the Atlantic group according to whether there was extensive snow lying or not.

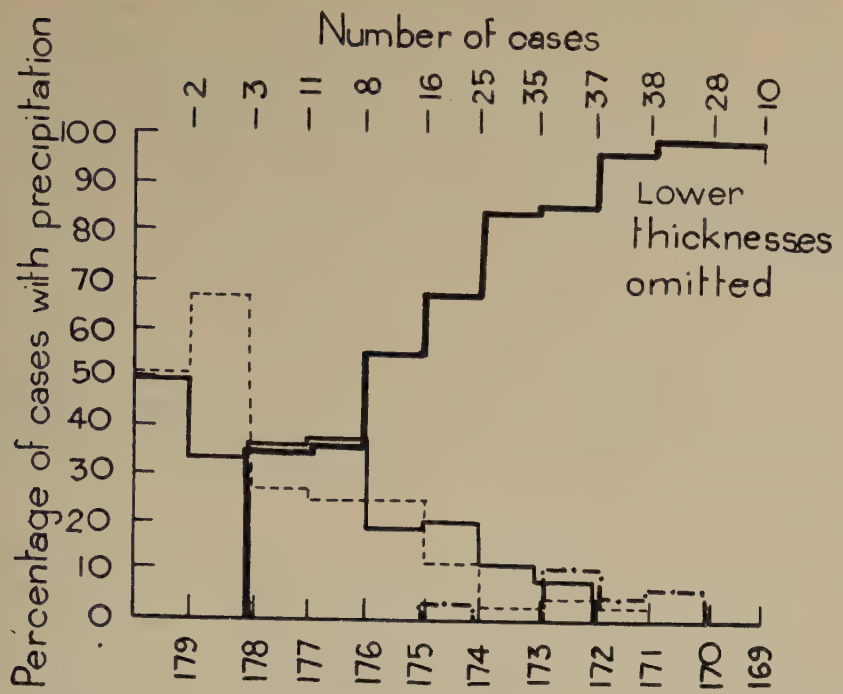

Figure 3. Type of precipitation and thickness value when snow is lying. Stations with snow lying: Oslo, 1.12.1950-15.4.1951; Stockholm, 4-11.12.1950 and 19.12.1950-9.4.1951; Riga, 7-10.12.1950 and 24.12.1950-6.4.1951; Berlin, parts of December, January and February 1950-51.

——— Rain, - - - - Drizzle, ——— Frozen forms (including sleet and hail), — · — · Showers (of all forms).

(The figures at top and bottom of this diagram are at the right-hand edge of the columns to which they refer).

Snow (or sleet) which fell with the higher thickness values, and all cases of snow with thicknesses over 17,700 ft, were associated with considerable inversions of temperature or deep isothermal layers and the precipitation may be presumed to have originated in a stratus or stratocumulus cloud-sheet below the stable layer. It is thought that in all cases with thicknesses of 17,700 ft or more the ground was already snow-covered. Cases which appeared to substantiate this as a general rule were noted in such unlike regions as Aldergrove (N. Ireland) and Oslo.

Data were insufficient for any thorough examination of variation with height of the liability for snow rather than rain with different 1,000-500 mb thickness values. The results for Eskdalemuir (about 800 ft above sea level) gave a more 'blurred' criterion (at least in the 1950-51 data here used) than for the low-lying stations examined and may have been as much affected by frequent occurrence of widespread snow lying over the Southern Uplands region in which the station lies as by its altitude.

(*b*) *Sleet*

Sleet was common only over a small range of thicknesses around the critical value for the time being for rain or snow. A few occurrences of sleet were noted with thicknesses as low as 17,100 ft at both Lerwick and Stockholm. Sleet may also occur with thicknesses up to 17,700 ft; but in the higher thickness ranges a clear-cut division between snow and rain is more usual, depending on whether the precipitation originates below the inversion or above it.

(c) *Hail*

Hail was rare or unobserved in winter at all the inland and continental stations. When it did occur it was only small hail, falling with any thickness values below 17,600 ft. On the other hand the larger forms of hail, such as occur in the Mediterranean area and Atlantic coasts in winter, and inland in the British Isles in summer, are known to occur with 1,000-500 mb thicknesses up to 18,000 ft or above.

(d) *Drizzle (including supercooled or freezing drizzle)*

Drizzle was observed with 1,000-500 mb thicknesses as low as 17,000 ft in the British Isles and 17,100 ft in the Baltic region. Presumably drizzle may occur at any temperature which is not too low to permit enough supercooled droplets of suitable size to form in low cloud or fog. The relative frequency of drizzle increased with increasing thickness (cf. Fig. 2 (*a*)).

(e) *Showers*

At Lerwick, Aldergrove and Eskdalemuir, where the influence of the Gulf Stream water was strongest, showers became more frequent the lower the thickness. At continental and inland stations reports of showers in winter were rare, especially when snow was lying, the December-February totals of showers at Mildenhall and Berlin being about one-tenth of the average for Lerwick, Aldergrove and Eskdalemuir (χ^2 test gives a probability well under one in a thousand of this difference occurring by chance). At Mildenhall and Berlin, moreover, showers appeared to be relatively most frequent with thicknesses about 17,200 ft. Lower thickness values were mostly associated with continental air which was either too dry or too stable in the lower layers to yield showery precipitation (it must be noted, however, that data on shower distribution with different thicknesses at the inland stations were too meagre to form the basis for a χ^2 test).

Table 5 gives a survey of the results for all the stations examined for the winter period December 1950-February 1951 and shows the main groupings.

TABLE 5. 1,000-500 MB THICKNESSES (IN FT) ACCOMPANYING RAIN OR SNOWFALL AT VARIOUS STATIONS (DECEMBER-FEBRUARY)

Station	Value giving equal likelihood of rain or snow	Highest thickness with snow	Lowest thickness with rain	Remarks
Lerwick	17,150	17,400	17,100	Sharp changeover
Eskdalemuir	17,550	17,700	17,100	No sharp criterion
Aldergrove	17,300	17,700	17,100	Fairly sharp changeover
Manchester	17,300	17,600	17,200	Fairly sharp changeover
Mildenhall	17,300	17,400	17,200	Fairly sharp changeover
Berlin	17,300	17,700	17,100	Fairly sharp changeover
Oslo	17,400	17,700	(17,300)	Less data in the transition region than for above stations
Stockholm	17,400	17,700	17,200	Sharp changeover
Riga	17,600	17,700	(17,300)	Data insufficient
Stations with snow lying	17,600	17,700	17,200	Changeover depends on whether precipitation originates below an inversion layer or not

(*f*) *Seasonal differences*

Monthly data for individual stations were generally too scanty to reveal anything significant. Various groupings, such as the monthly trends of critical thickness values at oceanic and continental stations, were looked at. No seasonal trend emerged, apart from an evident requirement of really low thicknesses (17,200 ft or below) for late spring snow and sleet in the British Isles from about mid-April onwards. This observation is doubtless related to rising sea-surface temperatures and increasing remoteness of the sources of cold air. Exceptionally early snowfall in autumn might be expected to demand similar conditions.

(*g*) *Geographical range*

The main differentiation noted under (*a*) was between oceanic and continental locations and as to whether a snow-cover already existed or not. Subject to this distinction the criteria for rain or snow (but not for hail) on low ground should have world-wide application.

As a check, 1,000-500 mb thicknesses over Malta, lying outside the normal limit of snowfall at the southern fringe of Europe, during winter 1952-3 were tabulated. The island is small and the surrounding water has a surface temperature in winter of about 13°C (55°F). The lowest thicknesses observed, 17,250 to 17,500 ft, were associated with cold rain and on several days in January with wet sleet falling at points over 200 ft above sea level. Snow was not observed. Hail-stones with up to 1 cm diameter fell with thicknesses ranging from 17,500 to 18,000 ft.

5. Association between the limit of an established snow or ice surface and 1,000-500 mb thicknesses in the atmosphere

A snow or ice surface has a very different effect upon the air over it from that of either dry or moist land or deep water. The diurnal range of temperature of the actual surface, whether of snow or of extensive sea-ice, is large and of the order of that of dry ground. The surface radiates out heat approximately as a black body, but a high proportion of incoming radiation is reflected and lost. The maximum temperature of the surface cannot in any case rise above 0°C until the snow or ice is removed, a process delayed by the requirement of much latent heat to be supplied from the air, from warm rain, from the underlying soil (or water) or by direct insolation.

Studies of ablation suggest that invasions of mild air over extensive snowfields in northern Europe in winter, typically accompanied by screen temperatures of the order of + 3°C (37°F) at a height of 4 ft above the snow, normally thaw about 1 in. of snow in 24 hr, if there is no rain or drizzle; the ablation increases to about 2 to 4 in./day if accompanied by appreciable rainfall (see, for instance, Champion (1950 and 1951)). Disappearance of 6 or 8 in. of snow may be expected to require either a continuous supply of mild air of several days' duration or a fall of an inch of rain or more.

Composite maps (once-daily isochrones of the snow or ice limit) – not reproduced – showing the position of the snow limit on successive days indicate that this limit behaves in such a way that rapid advances, often on a wide front, are followed by periods of stability or of more gradual retreat in which certain segments only of the snow margin are lost at any one time.

The rapid advances result from passage of a single depression or invasion of a single cold air mass giving instability showers. The process described in an earlier section whereby long-continued precipitation chills air that was formerly rather mild and dry

is also liable to result in important advances of the snow margin over previously bare ground.

Withdrawals of the snow margin follow all types of ablation. In central and northern Europe, including the British Isles, warm rain is the most effective agent for removing snow-cover in winter. At other seasons, and in regions south of about 45°N. even in winter, sunshine is most effective.

Positions of the snow limit which coincide more or less with a mountain barrier are more liable to persist, when warm air advances over the high ground (and commonly fails to descend to the low ground on the lee side), than positions anywhere over the European lowlands where no barrier exists. The effect is sufficiently obvious in the way the course of the mean 0°C surface isotherm over Europe follows the topography, with several successive weekly or monthly positions remaining close to the Norwegian and central European massifs. The greater frequency of snow lying everywhere over northern and eastern England east of the main watershed compared with the same levels on the western side (Manley 1947) is also at least partly attributable to this cause.

Extensive lying snow cools warm air passing over it in a manner largely similar to the case of warm air in summer over the Arctic ocean and other cold water regions (downward eddy-flux of heat) (cf. Taylor 1914, Sverdrup 1936, Scherhag 1937). Over winter snow there may be, additionally, important radiation cooling from the top of fog and stratus formed in the mild air. Where the snow limit coincides with a mountain barrier, any forward thrust of warm air may be lifted above the surface and is liable to contribute frontal and orographic snowfall which adds to the depth of the lying snow. These effects modify the air and limit the freedom of movement of the thickness isopleths.

The writer therefore decided to investigate the occurrence of various 1,000-500 mb thicknesses over established snow and ice and in the neighbourhood of the snow limit. Positions of the snow limit over Britain and Germany were obtained by mapping actual state-of-ground reports. The number of cases in which the snow limit was accurately known in this way was augmented, for the purpose of the tests, by many other cases in which the position of the limit could be deduced beyond reasonable doubt, to within 100 mi, from consideration of the reported sequence of weather for a week or more and the behaviour of the surface air temperatures. The last-named are particularly revealing whenever a broad warm air stream encounters lying snow and a stationary belt of sharp fall of surface temperature to + 3°C (37°F) or lower* in the moving air stream marks the snow limit. This reasoning frequently made it possible to place the snow margin in many parts of Europe and N. America. Composite maps were then drawn showing successive daily positions of the limit of snow and ice.

(*a*) *Thickness values at the limit of snow- and ice-covered surface*

For this test, reported 1,000-500 mb thicknesses and values estimated from the isopleths were taken at points over the western and south-western limit of contemporary snow-cover in Europe between 45° and 60°N, care being taken to choose points where the snow edge was fairly straight and free from deep embayments. Points used were in west Norway, western Britain, France, Germany, Poland or Sweden in a region well covered by upper-air observations. Over 80 per cent of the points chosen were on ground less than 300 ft (100 m) above sea level, the remainder being at the edge of a mountain

* It should perhaps be made clear that this does not mean that the 3°C isotherm (or any other value) is used as a criterion for defining the snow limit. Rather is it the strong gradient of surface temperature which is used and, even then, only those cases which indicate the snow limit within obviously narrow limits of error have been accepted.

barrier; in neither case did it appear necessary (either *a priori* or in the light of results obtained) to make any allowance for the elevation of the actual test point, so long as all the low ground (below 300 ft) to leeward was snow-covered. The region studied was also entirely within the zone of prevailing westerly winds aloft : any association found between the thickness value and the windward limit of snow-covered land (in the sense of the prevailing upper westerlies) would therefore be liable to imply some distortion of the atmospheric circulation aloft.

The mean value of 1,000-500 mb thickness given by 1,053 observations over the best attested positions of the snow limit in this European region on 99 different days between January 1947 and February 1952 was 17,297 ft, the standard deviation being about 170 ft.

This figure should be compared with normal thicknesses between January and early March in different parts of the region studied ranging from about 17,200 to 17,700 ft with a S.D. at any given place of the order of 300 ft (e.g., mean thicknesses for Larkhill for two successive Januarys were respectively 17,460 ft, S.D. 523 ft and 17,657 ft, S.D. 259 ft. A variance ratio test, comparing the variance at the snow limit with that at Larkhill in both years, shows that the difference exceeds the 0·1 per cent chance probability-level).

The low mean thickness and small standard deviation at the snow limit, wherever that may happen to be, point to a real association between the snow limit and the 1,000-500 mb thickness over it and to a contemporary shift of the upper-wind pattern (thermal and actual).

Various additional tests were made to broaden the basis of this result and to gain some indication of its application to other geographical regions.

The average 1,000-500 mb thickness, from eighty-five radiosonde ascents made by the Dutch whaling ship *Willem Barendsz*, near the pack-ice in 60°S between 12°E and 30°W (December 1946-April 1947) in the South Atlantic was 17,260 ft, S.D. 212 ft. The standard deviation might reasonably be supposed to be a little less if taken precisely at the ice margin in all cases. (The difference from Larkhill, even in this sample, however, approaches the 1 per cent chance level of significance when tested by variance-ratio test.)

As no further actual positions of the snow or ice limit were readily available, tests were made of the mean 1,000-500 mb thickness values at points on the normal position of the M.S.L. 0°C isotherm shown by the official *Russian Climatological Atlas* (published 1930) for Russia and Siberia and by Bartholomew's *Atlas of Meteorology* (published Edinburgh 1899) for North America. The Russian and Siberian material supplied mean positions for the 0°C isotherm at about 10-day intervals from 11 September through the winter to 1 June. The North American material yielded only monthly mean positions, which were taken as applying to the middle day of each month. The means related to the years 1849-1899 for Siberia, 1881-1915 for European Russia and 1870-1884 for N. America. An obvious weakness was that the mean positions of the 0°C isotherm at sea level for these years had perforce to be compared with mean 1,000-500 mb thicknesses about the same calendar dates for the period 1946 to 1950. In practice, thickness means for seven days centred upon the relevant dates were taken at each intersection of the 0°C isotherm with the meridians of longitude, every ten degrees of longitude from 120°W to 60°W and from 20°E to 140°E. This yielded 3,268 values. The mean was 17,507 ft and the standard deviation 280 ft.

Considering the crudeness of the test, this result agrees about as closely as we could expect with the values obtained for the actual snow limit on the actual day and year. The discrepancy is likely to be due chiefly to two causes :

(i) The 0°C mean isotherm need not by any means coincide with the snow limit for a given period, but seems likely to protrude over bare ground somewhat beyond the snow-covered region.

(ii) The snow-cover has probably been rather less extensive in recent years, apart from the winter 1946-7 in the European sector, than in the nineteenth century.

From these tests it seems fair to conclude that the mean 1,000-500 mb thickness over the contemporary windward limit of snow and fast ice on the polar seas is close to 17,300 ft in all parts of the world and that the variation of thickness over the snow margin is distinctly smaller than that at any place well away from the margin.

(*b*) *Movement of warm air and incursions of various thickness isopleths over extensive snow and ice-fields*

Incursions of particular isopleths of 1,000-500 mb thickness were measured in nautical miles along the path of the gradient wind. Choice was restricted to cases where the snow limit was stabilized in a given position for a period of seven days or more. Note was then made of the *maximum* incursion of a given thickness isopleth along the instantaneous line of surface gradient wind, the mean strength of the gradient wind along that line at the time in question, the nature of the terrain and type of weather situation.

The measurements of maximum encroachment of various thickness values above 17,300 ft over a widespread snow- and ice-covered terrain are fully logged in Table 6, (the greatest encroachments were commonly maintained at more or less the extreme distance for several days by persistent advection of warm air during the life of the *Grosswetterlage*; brackets indicate extreme penetrations which were notably short-lived). All cases which may have been complicated by subsidence (deduced from anticylonic curvature of the isobars and appropriate surface weather reports) are separately listed. Thirty-eight 7-day periods taken from six winters between 1 Jan. 1947 and 29 Feb. 1952 provided the required conditions for test measurements in different parts of the northern hemisphere including Alaska, Canada, Europe and western Siberia.* In intermediate cases the distance of greatest encroachment of a given isopleth over snow has been rounded upwards to the next hundred miles, whereas gradient wind (V_G in the table) has been rounded downwards to the next lower unit in tens of knots. This means that, in spite of the rather small number of cases available for study, there is an enhanced likelihood of an observation as logged in the table approximating to the greatest incursion possible. Results from the first five winters (30 weeks in all permitting 35 measurements) were first logged as a separate group and analysed graphically to discover likely asymptotes (maximum incursions and minimum gradient winds); thereafter results for the sixth winter (8 weeks in 1951-2, permitting 12 measurements) were similarly treated and compared. The additional material from the sixth winter (about 30 per cent of the original data considered) repeated the same pattern and pointed to substantially the same extremes.

We conclude from this table that :

(i) Thicknesses of 18,000 ft do not transgress at all over extensive snowfields without the aid of subsidence.†

(ii) Thicknesses of 17,800 ft do not encroach more than about 500 nautical mi over snow without the aid of subsidence.

* The terrains covered were of course infinitely varied and difficult to summarize. Maximum encroachments, measured either over extensive lowland (e.g., the north European plain) or over an *initial* mountain barrier which also marked the contemporary snow limit (e.g., the Alps and Alpine foothills) were, however, found to give a reasonable gradation with wind speed (Tables 6 and 7) without making any allowance for the elevation of the country.

† 18,000 ft is believed to be also about the maximum thickness observed over the inner Arctic basin in summer at the time of greatest loosening of the pack-ice on the polar sea.

TABLE 6. GREATEST ENCROACHMENTS OF VARIOUS 1,000-500 MB THICKNESSES OVER SNOW AND ICE SURFACES, MEASURED ALONG GRADIENT WIND (all cases)

Gradient wind at M.S.L. (mean strength along path) (kt)	Distance of greatest incursion over snow or ice surface in nautical miles (hundreds) (Tabulated figures below are numbers of cases; each case the greatest incursion in 7 days)																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Incursions of the 17,400 ft isopleth (47 cases in all)																								
70						1																				
60			1							2						(1)										
50				1						1																
40				1	2		3	1 1*	2			(1*)	(1)			1*										
30	1		1	1			1			2	((2))														(1*)((1*))	
20	2	1*	1*		1						1*		1*				((1*))									
10																										
0	3	1	1*	2			1*																			
	Incursions of the 17,600 ft isopleth (45 cases in all)																								
70					1																					
60	1			1			1					(1)														
50	1					1																				
40	1 1*	1	1	1	3 1*	2					(1*)		1*								(1*)		(1*)			
30	3		1	1	1	1*				2*																
20		1 1*							2*																	
10																										
0	5 1*	1				1*																				
	Incursions of the 17,800 ft isopleth (39 cases in all)																								
70	1																									
60	2				1	(1)																				
50	2																									
40	6 2*	1		1	(1*)						1*															
30	4 1*								1*																	
20	2 2*		1*	1*																						
10																										
0	6 2*																									
	Incursions of the 18,000 ft isopleth (36 cases in all)																								
70																										
60	4																									
50	2																									
40	7 3*					((1*))																				
30	3 2*																									
20	1 4*																									
10																										
0	6 2*																									

Cases complicated by subsidence, indicated by anticyclonic curvature, are shown with an asterisk. ( ) means maximum incursion maintained for less than 48 hr. (( )) means maximum incursion lasted under 12 hr.

(iii) For thicknesses of 17,400 ft the maximum incursion is some 1,500 nautical mi without subsidence, but may reach 2,500 mi or more for a short time when subsidence is occurring.

In all cases the final limitation on greater incursions appears to be imposed by the improbability of gradient winds of the requisite strength being maintained over the necessary distances.

Table 7 lists the *maximum encroachment* of the various thickness isopleths indicated by graphical analysis of the cases in Table 6 (figures in brackets are rather free estimates; hyphens have been entered where no estimate was justified by the data available). The effect of subsidence or its absence may be clearly seen. A third column shows the result of singling out cases where the snow limit was stabilized near a mountain barrier athwart the path of invading warm air.

Coincidence of the snow limit with an orographic barrier (which need not be more than 1,000 m in height) may have a decisive effect. The right-hand column in Table 7 includes cases of snow limits stabilized at the Alps and Rocky Mountains, at the Norwegian highlands and in two cases at the Pennine and Welsh hills in Great Britain. In no case was there any incursion at all, even of the 17,400 ft thickness isopleth, over the snow with less than 30 kt winds carrying warm air over the barrier; maximum encroachments were enormously reduced even with a major thrust of warm air and high values of mean gradient wind.

TABLE 7. MAXIMUM ENCROACHMENT OF GIVEN THICKNESS LINES OVER EXTENSIVE SNOW AND ICE, MEASURED IN NAUTICAL MILES ALONG THE AIR STREAM (from graphical analysis of the figures in Table 6)

V_G (kt)	Cases with subsidence	All cases without subsidence	Cases without subsidence where snow limit was stabilized near a topographical barrier
		17,400 ft thickness	
0	800	250	0
15	1,200	500	0
30	1,800	1,000	130
45	2,400	1,200	600
Abs. max.	2,400	1,500	900
		17,600 ft thickness	
0	500	under 200	0
15	900	250	0
30	1,500	450	under 100
45	2,200	750	–
Abs. max.	2,200	1,100	500
		17,800 ft thickness	
0	250	0	0
15	450	under 200	0
30	900	200	0
45	(1,400)	350	under 300
Abs. max.	–	500	300
		18,000 ft thickness	
0	0	0	0
15	(200)	0	0
30	(400)	0	0
45	(600)	0	0
Abs. max.	–	0	0
Numbers of cases used	11	24	12

Checks upon the statistical significance of the grouping of data in Table 7 into cases with subsidence and without, and where a barrier was operative, showed (t tests) that the *average* encroachments of the 17,400 ft thickness line were significantly different in the three types of case (chance probability 1 to 2 per cent). There was no significant difference of variance ratio between the subsidence cases and the others, but the smaller variance of the barrier cases had a less than 5 per cent probability of being attributable to chance.

(c) *Advance of cold air and movement of various thickness isopleths over bare ground and ocean waters beyond the snow limit*

The warming of cold air over the sea has been studied by Craddock (1951) and others, and their work need not be repeated here.

Movement of the thickness isopleths for values below 17,300 ft beyond the snow and ice margin was however briefly studied. Such movements over land are nearly always accompanied by advances of the snow limit itself. Over open ocean no such rapid changes in the distribution of frozen surface are possible. Indeed at certain times forward thrusts of cold air accompanied by high, stormy winds may cause a good deal of ice to break adrift and bring about some withdrawal of the effective ice margin.

Examination of the movement of various thickness isopleths over the oceans north of 50°N yielded the following results :

(i) Movement of the 17,400 and 17,200 ft isopleths appears unlimited except in the southerly direction.

(ii) Extreme advances of the 17,000 ft isopleth over the ocean may be limited to about 1,000 nautical mi with extreme winds south of latitude 60°N but are apparently unlimited farther north.

(iii) 16,800 ft thickness was not observed more than 800 nautical mi from the ice margin, except north of latitude 60° where greater advances over open water occur with high winds.

(iv) 16,600 ft and lower thickness values were not observed at all away from the ice, save with winds of over 50 kt and in latitudes north of 60°N.

(v) The 16,200 ft isopleth in no circumstances advanced more than 400 nautical mi from the ice.

These conclusions are based on tests in 33 different weeks of clearly identified ice margin during six winters (1947-52).

6. Summary of results and concluded implications

(a) *Results*

(1) We have found a fairly regular changeover of precipitation from unfrozen to frozen forms with 1,000-500 mb thickness about 17,300 ft. Over already established snowfields the normal thickness value for the changeover is higher, about 17,400 to 17,600 ft. Over seas and oceans with surface water temperature about 10°C the average thickness value for the changeover from rain to snow is lower, about 17,150 ft. In extreme cases the changeover may come with any value from 17,100 to 17,900 ft, but in most circumstances 17,300 $\pm$ 150 ft (5,280 $\pm$ 45 m) is the critical thickness (the variations depend greatly on lapse rate of temperature in the lowest 1 to 2 km, but also on whether the precipitation is formed in the cloud masses of the friction layer or at higher levels).

(2) The average 1,000-500 mb thickness at the windward limit of extensive snow and ice is also found to be just 17,300 ft with a standard deviation only about two-thirds of that characteristic for specific points normally far away from the snow or ice margin.

(3) Movement over extensive snow of the thickness isopleths for values of 17,400 ft and higher is subject to certain restrictions. Many sorts of cooling processes are involved.

(4) When the windward limit of the snow is protected by an orographic barrier from invading warm air, advance of the higher thickness values is much more restricted. Correspondingly, in these cases, the snow-cover is more likely to persist and continue to restrict the movement of the upper air pattern. Invading warm fronts are also liable to be held up and occluded at the snow margin where this is fortified by mountains (on occasion even the hills of the British Isles) – a process which commonly yields more snow.

(5) Warming over open sea sharply restricts the movement of isopleths of the lower thickness values, when cold air thrusts out beyond the snow or ice limit. Similar forward thrusts of cold air over land usually spread the snow-cover itself to new limits.

(6) The evidence suggests that extensive lying snow itself plays some part in maintaining the cold trough in the upper troposphere which favours persistence of the snow. A snowfield of 1,500 nautical mi west to east extent – i.e. 50° of longitude in 60°N or 35° of longitude in 45°N – would be sufficient, while it lasted, to cause a trough in the 17,400 ft isopleth of 1,000-500 mb thickness, unless advection of warm air at an average velocity of 50 kt or more were maintained across the entire 1,500 mi or appreciable subsidence were at work. In most cases a *broad* trough in the thickness pattern would result. Moreover, consideration of the distribution of normal thickness values over the northern hemisphere shows that in mid-October, when the first extensive snow-deck is established over much of north-eastern Canada, and over eastern Asia north of the Mongolian plateau in 50-55°N, the associated cold troughs should bring the thickness down to 500 to 800 ft below the values at that time ruling over the (eastern) North Atlantic and (eastern) North Pacific oceans in the same latitude. As the winter progresses, these major cold troughs grow in longitudinal extent and in intensity and are thrust farther south, until widespread snow reaches latitude 40° to 45°N over parts of N. America and Asia in January.

These results have an obvious bearing upon the much debated question of the primacy of thermal or orographic influences in the general circulation of the atmosphere (see, for instance, Sutcliffe 1951). The investigation itself has provided instances of the effectiveness of both these factors as partial controls acting upon the circulation in the upper troposphere.

(*b*) *Further implications*

Several corollaries throw new light on known facts of meteorological experience :

(1) For the occurrence of exceptionally prolonged and severe winters in the British Isles, the development of a continuous snow-cover right across Europe north of the Alps and Carpathians (and abnormal cooling of the North Sea surface) must be important. A snow mantle over the British Isles and western fringe of Europe alone, such as sometimes results from outbursts of maritime arctic air from the Norwegian Sea, has far too little longitudinal extent to exercise any decisive cooling upon the atmosphere or appreciably to restrict the eastward advance of a procession of ridges and troughs in the upper westerlies. Cases of this occurred in 1944-5, 1951-2 and 1952-3, and the snow abruptly disappeared in mid-winter, followed by persistently mild westerly spells.

(2) Baur (1951, 1954) has found that for Berlin, between 1848 and 1952, winters in which the first ten days of December had a mean temperature at least 2·5°C above

normal were invariably mild winters as a whole and that the mean temperature of the following January and February was above normal or, at all events, near normal (in this period there were some 21 cases : three of those winters did, however, contain spells of respectively 9, 16 and 24 days of severe weather at Berlin, although the stipulated condition was fulfilled and the mean temperature of the winter as a whole was above normal). Baur further attributes the mild conditions of early December, which presage a generally mild winter, to south-westerly or westerly winds reaching central Europe and recurring at intervals throughout the winter : this stipulation has acquired all the more interest because 1-10 December 1953 was 4°C above normal in Berlin, though associated with a persistent mild southerly (or south-easterly) situation, and the ensuing winter was notably severe – the first apparent failure of the Berlin rule for over 100 years. For prolonged severe weather (lasting much over three weeks) in western Europe it is likely that the development of a widespread snow-sheet across Eurasia to near eastern Germany by early December is a necessary, though not a sufficient, condition (this condition would hardly be fulfilled in predominantly westerly or south-westerly situations, but it *was* fulfilled in the much more unusual situation of December 1953).

(3) A broad continental snow-cover, as over the northern continents in winter, must favour an upper-air pattern with a great warm ridge near, and just outside, the western limit of the snow and a broad cold trough over the snow. Variations of detail on this basic upper-air pattern are likely to produce, repeatedly, situations favouring anticyclogenesis near (and over) the north-western portion of the snow-covered region and cyclogenesis just outside the eastern or south-eastern limit.

(4) An established continental snow-cover in any winter may be expected to encourage the steering of depressions along its perimeter, including south-eastward-moving systems near the western limit of the snow, cf. also attempted reconstruction of ice-age phenomena by Simpson (1934) and Manley (1951).

(5) Restrictions imposed by extensive snowfields on free movement of the thickness isopleths must limit the possibilities of any radical change of depression tracks unless and until a sufficiently vigorous pulse of warm-air advection removes the snow.

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Low-temperature fog at the Edmonton Airport as influenced by moisture from the combustion of natural gas

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Summary

The Edmonton Municipal Airport, being within the city limits, is completely surrounded by a source of moisture from the combustion of natural gas. Since the natural gas is metered to consumers, a quantitative study of the injection of moisture into the air around the airport is possible. Equations and graphs show how moisture in the air at the airport is influenced by temperature, wind velocity and initial relative humidity. The results obtained are compared with fog conditions observed at the airport during the winter of 1949-50.

1. Introduction

The Edmonton Municipal Airport located two and a half miles north-west of the city centre is surrounded by residential areas and industrial plants. The water vapour produced by the combustion of natural gas in heating plants poses a problem in fog prediction, particularly at low temperatures during the winter.

Ceiling and visibility conditions are normally exceptionally good; natural gas burns without smoke and since it is the main fuel in Edmonton, local smoke rarely reduces visibilities below 3 mi. During extremely cold spells, however, when saturation vapour pressure is low and when the production of moisture by combustion is high, extended periods of poor visibilities in fog or ice fog are occasionally experienced.

The winter of 1949-50 was the second coldest at Edmonton in nearly 70 yr, affording favourable conditions for the formation of local fogs of varying intensities. During the period of this investigation, 10 Dec. 1949 to 31 Jan. 1950, the temperature dropped to 0°F or lower for 725 hr and on one occasion touched — 48°F. Visibility was 0-$\frac{1}{8}$ mi in ice fog for 40 hr and 3 mi or less for 197 hr. The longest continuous period with visibility 3 mi or less was 36 hr and for 22 hr of this period it was $\frac{1}{8}$ mi or less.

2. Factors influencing fog at low temperatures

Four factors to be considered which may influence the formation or absence of fog or ice fog are : (a) temperature, (b) wind direction, (c) wind speed and (d) moisture supply. The regular hourly observations were abstracted in such a manner as to show the visibilities which occurred at various wind speeds, wind directions and temperatures. Only those cases where the visibility was unrestricted or where it was restricted by fog, ice fog and ice crystals, or combinations of any of these obstructions with smoke, were used. Fogs at Edmonton are very rare at temperatures above 0°F so this study was confined to temperatures below zero.

3. Discussion of effects of wind and temperature

(*a*) *Visibility and temperature*

There were relatively few cases, as shown in Table 1, with the visibility below 6 mi when the temperature was above — 20°F. Below this the visibility becomes progressively lower until at — 40°F it is usually $\frac{1}{8}$ mi or less.

TABLE 1. DURATIONS OF OCCURRENCE IN HOURS OF VARIOUS VISIBILITIES FOR VARIOUS TEMPERATURE RANGES BELOW 0°F, EDMONTON AIRPORT (total number of hours = 725)

Temp. (°F)	Visibility in miles								
	0 to $\frac{1}{8}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{3}{8}$ to $\frac{1}{2}$	$\frac{5}{8}$ to $\frac{3}{4}$	$\frac{7}{8}$ to 1	$1\frac{1}{4}$ to 2	$2\frac{1}{4}$ to 3	4 to 6	over 6
0 to − 4						1	1	3	88
− 5 to − 9			1			2	5	7	60
− 10 to − 14					1	3	2	21	75
− 15 to − 19						5	6	15	82
− 20 to − 24					2	7	16	16	66
− 25 to − 29				5	5	13	11	29	24
− 30 to − 34	2	2	5	6	4	19	6	17	22
− 35 to − 39	8	6	8	5	3	4		3	
− 40 to − 44	20	1	1						
− 45 to − 49	10	1							

Visibilities are reported for certain specified distances only. All reported distances are included in the ranges as given here and in following tables.

Low visibility accompanying extremely low temperature has been observed in other countries. Frost (1934) noted that, in Alaska, fog was always associated with extreme cold and that, at Fairbanks, dense fog always accompanied temperatures of − 45°F or lower. Petterssen (1940) found for stations in Siberia that there was a tendency for fog frequency to increase with decreasing temperature to − 56°F. Below this there was the suggestion that the fog frequency decreases.

To indicate that the relationship between visibility and low temperature at Edmonton is more than a result of the low saturation vapour pressure, hourly data from a rural airport at Vermilion were analysed for the same period. This airport lies in open flat country 120 mi east of Edmonton and about one mile east of the town of Vermilion. The meteorological regime of the two sites should be very similar with the exception of the source of moisture from combustion. The population of Vermilion was about 1,900 at this time whereas that of Edmonton was 159,000, so that the consumption of natural gas at Vermilion should be at most only 1·2 per cent of that at Edmonton.

TABLE 2. DURATIONS OF OCCURRENCE IN HOURS OF VARIOUS VISIBILITIES FOR VARIOUS TEMPERATURE RANGES BELOW 0°F, VERMILION AIRPORT (total number of hours = 773)

Temp. (°F)	Visibility in miles								
	0 to $\frac{1}{8}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{3}{8}$ to $\frac{1}{2}$	$\frac{5}{8}$ to $\frac{3}{4}$	$\frac{7}{8}$ to 1	$1\frac{1}{4}$ to 2	$2\frac{1}{4}$ to 3	4 to 6	over 6
0 to − 4								2	43
− 5 to − 9								1	79
− 10 to − 14									84
− 15 to − 19					2			1	105
− 20 to − 24					2			2	118
− 25 to − 29					1		3	2	102
− 30 to − 34			1		2	1	3	3	57
− 35 to − 39	1	3	4	2	2	3		1	40
− 40 to − 44	1	1	4	2	2	2	1		25
− 45 to − 49	1	3	1	1	3	4		1	24
− 50 to − 54	2	8		2	3		1		3
− 55 to − 59		3			2		1		

The effect of moisture from the combustion of natural gas on visibilities is obvious when the visibility-temperature distributions for Edmonton (Table 1) and Vermilion (Table 2) are compared. At temperatures below − 40°F there were 30 out of 33 occasions when the visibility was $\frac{1}{8}$ mi or less at Edmonton whereas on only 4 out of 103 occasions was the visibility so low at the rural site.

TABLE 3. DURATIONS OF OCCURRENCE IN HOURS OF VARIOUS VISIBILITIES FOR VARIOUS WIND DIRECTIONS (total number of hours = 725)

Wind direction	Visibility in miles								
	0 to $\frac{1}{8}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{3}{8}$ to $\frac{1}{2}$	$\frac{5}{8}$ to $\frac{3}{4}$	$\frac{7}{8}$ to 1	$1\frac{1}{4}$ to 2	$2\frac{1}{4}$ to 3	4 to 6	over 6
S-SSW	31	4	7	6	4	17	8	28	108
SW-WSW	6	3	5	5	3	7	9	18	38
W-WNW		3		3	2	6	6	23	105
NW-NNW			3	2	3	15	14	26	99
N-NNE	3					5	1	2	29
NE-ENE					3	3	5	6	22
E-ESE							1	3	4
SE-SSE						1	3	5	12

TABLE 4. DURATIONS OF OCCURRENCE IN HOURS OF VARIOUS WIND DIRECTIONS FOR VARIOUS TEMPERATURE RANGES BELOW 0°F (total number of hours = 725)

Wind direction	Temperature range °F		
	0 to − 19	− 20 to − 34	− 35 to − 50
S-SSW	78	94	41
SW-WSW	33	40	21
W-WNW	83	60	5
NW-NNW	103	59	
N-NNE	35	2	3
NE-ENE	23	16	
E-ESE	4	4	
SE-SSE	19	2	

(*b*) *Visibility and wind direction*

Visibilities of one mile or less, as indicated in Table 3, occur most frequently with winds from the S., SW., W. and NW. Visibilities near zero occur most frequently with winds from the S. and SSW. Winds from the NE. and SE. occur rather infrequently at low temperatures as shown in Table 4. Below − 34°F the predominating direction is almost uniquely from the S. or SW. The absence of south-easterly winds is accounted for by drainage down the natural slope of the airport and the surrounding terrain towards the south-east. On the airport alone there is a 20 ft drop along the NW.-SE. runway.

TABLE 5. DURATIONS OF OCCURRENCE IN HOURS OF VARIOUS VISIBILITIES FOR VARIOUS WIND SPEEDS (total number of hours = 725)

Wind speed (mi/hr)	Visibility in miles								
	0 to $\frac{1}{8}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{3}{8}$ to $\frac{1}{2}$	$\frac{5}{8}$ to $\frac{3}{4}$	$\frac{7}{8}$ to 1	$1\frac{1}{4}$ to 2	$2\frac{1}{4}$ to 3	4 to 6	over 6
0 to 1	5			1	1	11	5	14	14
2 to 3	8	4	7	9	5	16	20	36	72
4 to 5	14	4	7	3	4	9	10	25	66
6 to 7	12	2	1	2	3	8	7	13	67
8 to 9	1			1	1	6	4	12	88
over 9					1	4	1	11	110

(*c*) *Visibility and wind speed*

At wind speeds greater than 7 mi/hr visibilities are generally over one mile as shown in Table 5. Visibilities near zero are more frequent with a wind of 4 or 5 mi/hr than with lighter winds. Calm or near calm winds transport water vapour so slowly from the city source that much of it sublimes on snow or falls as ice crystals before reaching the airport in concentrations sufficient to restrict the visibilities to the lowest values.

TABLE 6. RELATIVE FREQUENCIES OF OCCURRENCE OF VISIBILITIES EQUAL TO OR LESS THAN SPECIFIED VALUES AT CONSECUTIVE PERIODS DURING THE DAY

Visibility (mi)	Time MST 0030 0130	0230 0330	0430 0530	0630 0730	0830 0930	1030 1130	1230 1330	1430 1530	1630 1730	1830 1930	2030 2130	2230 2330
≤ 6	29	31	32	34	33	33	18	10	16	21	24	24
≤ 3	18	20	19	20	31	29	12	5	8	11	12	15
≤ 1	11	9	9	8	19	11	3	2	4	5	6	7
≤ ½	7	8	6	7	14	7	2	1	3	2	3	2

(*d*) *Diurnal variation of visibility*

A maximum frequency of low visibilities occurs at 0830 and 0930 MST as is shown in Table 6. There are four main reasons for this :

(i) Minimum temperature usually occurs during this period and consequently the vapour-pressure deficit is at a minimum.

(ii) Wind speeds are lowest during this period.

(iii) Heating systems, which were turned low overnight, are turned up nearly to full during or immediately prior to this period, producing a maximum supply of water vapour.

(iv) Just after daybreak (sunrise at 0842 MST, 31 Dec.) local flying and unscheduled flights begin operation. If the temperature and vapour pressure are at a critical value, each departing aircraft leaves a condensation trail on the runway which may quickly envelop a large part of the airport. The departure of two or three aircraft in quick succession has been observed to create sufficient fog to lower the visibility from 5 or 6 mi to near zero when the temperature is very low.

The minimum frequency of occurrence of low visibilities is at 1430 and 1530 MST, when wind speed, turbulence, temperature and vapour-pressure deficit are at a maximum and the moisture supply is at a minimum.

4. FAVOURABLE SYNOPTIC SITUATION

Fogs which come under the classification of this discussion occur in polar or arctic air masses. The highest temperatures and strongest winds occur in the shallow forward portion of such air masses. As the air mass deepens and the polar high or ridge passes over or to the east of Edmonton, skies clear and radiational cooling causes temperatures to drop rapidly. Simultaneously, winds drop to light and veer into the south or south-west, making conditions ideal for extremely low temperatures and ice fog.

A typical synoptic situation is illustrated in the chart of Fig. 1. At the time of this chart Edmonton reported a temperature of $-45°$F, with the visibility down to $\frac{1}{8}$ mi. Other nearby stations reported temperatures as low as $-55°$F, but visibilities were from two miles to unrestricted. This suggests that the large artificial supply of moisture from fuel combustion at Edmonton is an important factor in the formation of local fog.

5. MOISTURE SUPPLY

(*a*) *Moisture supply and temperature*

In order to determine a relationship between gas consumption and temperature, daily total gas-consumption amounts for the city were obtained from Northwestern Utilities Limited for December 1949 and January 1950. These data, together with the corresponding mean daily temperature, were divided into 6 groups according to temperature

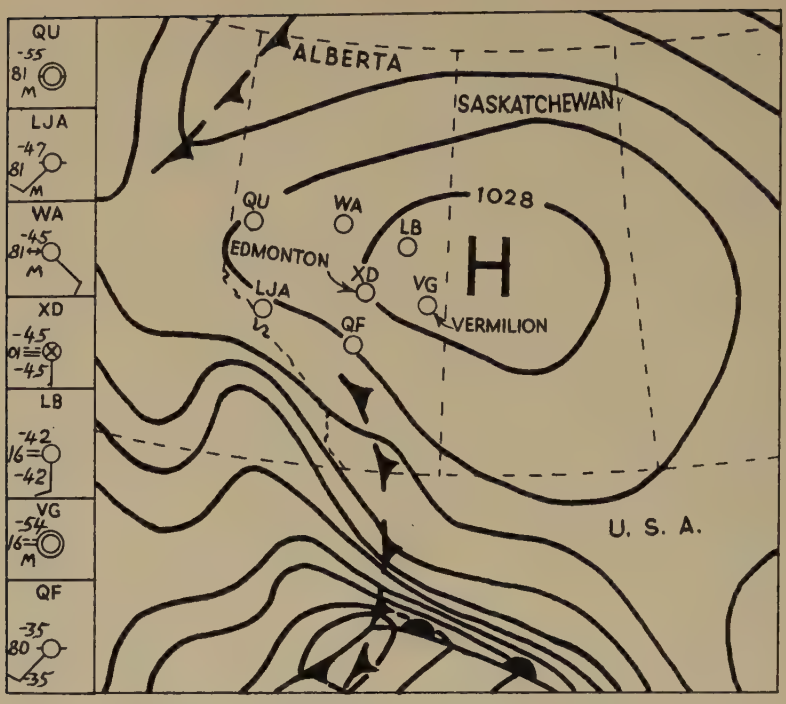

Figure 1. Synoptic chart 1230 GMT, 14 Jan. 1950.

(*Added in proof* : from the triple point northward, the spikes of the front should be reversed).

range and averages for each group calculated. The resulting six pairs of values were plotted on a graph, Fig. 2. The amount of fuel used should be proportional to the number of day-degrees. The gas-consumption curve tails off, however, at temperatures below — 16°F because not all heaters are designed for the lowest possible temperatures. Thus as temperatures decrease further the number of heaters reaching maximum consumption of gas steadily increases. The equation :

$$G = -926000 + 9190T - 20{\cdot}7T^2 \quad . \quad . \quad . \quad (1)$$

where G = consumption in $10^3 \times \text{ft}^3$/day and T = mean daily temperature in °K, was found to give a satisfactory relationship for the temperature range from 50°F to — 50°F.

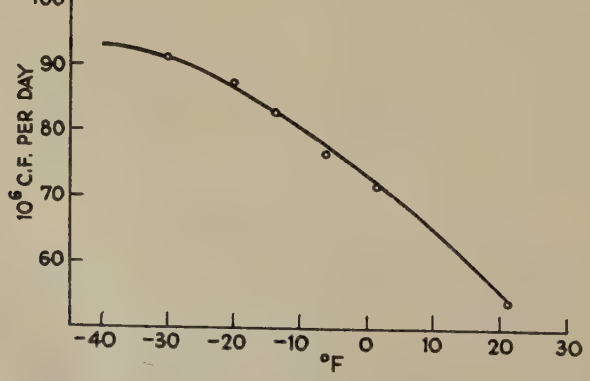

Figure 2. Gas consumption as a function of temperature.

One cubic foot of natural gas (chiefly methane) produces, upon combustion, 42·64 of water vapour. Thus, it is possible to calculate the average time required to saturate initially dry air over the city to a given arbitrary depth. This time, in arbitrary units, given by :

$$t_1 = \frac{ADw\rho}{42{\cdot}64\,G} \qquad . \quad . \quad . \quad . \quad (2)$$

where A is the gas-consuming area of the city = 17·60 mi^2

D is the depth of the saturated air (ft)

w is the saturation mixing ratio = exp (22·4 − 5712/T) g/kg (applicable to low temperatures in the lower layers of the atmosphere)

ρ is the density of the air g cm^{-3} = P/2·87 × 10^3 T (P = pressure in mb).

Substituting Eq. (1) in Eq. (2) and using the above values for A, w, and ρ, the time (in hours) required to saturate initially dry air to a depth of 100 ft can be expressed in terms of temperature only :

$$t_1 = \frac{25{\cdot}4 \times 10^7 \exp\,(22{\cdot}4 - 5712/T)}{T(-926000 + 9190T - 20{\cdot}7T^2)}$$

assuming a mean surface pressure P of 930 mb.

If the moisture is diffused or carried upwards through a depth, D, and if the initial relative humidity of the air is U, then the time required to reach saturation is,

$$t = t_1 \frac{D}{100} \frac{100 - U}{100}$$

Curves showing the relationship between time required for saturation, temperature, and various initial relative humidities are shown in Fig. 3. The depth of the air considered is 100 ft for all curves. The graph shows that air initially at − 20°F, and 85 per cent relative humidity, would be saturated to a depth of 100 ft in only 0·7 hr. As the temperature

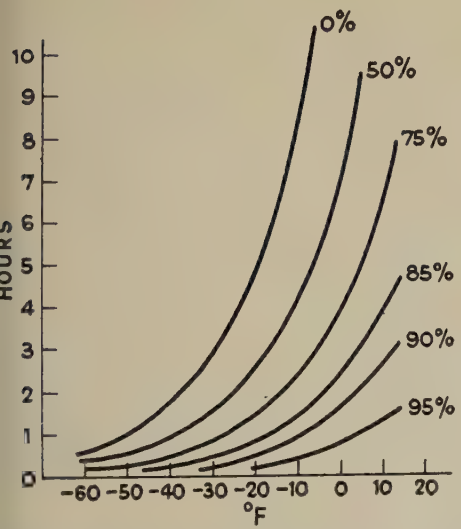

Figure 3. Time required to saturate still air over Edmonton at various temperatures and initial relative humidities.

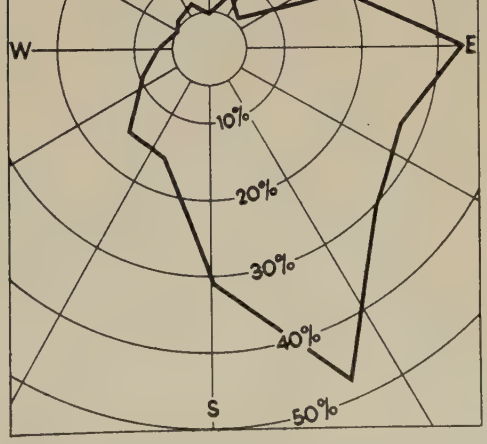

Figure 4. Gas consumption in various directions from the airport in terms of the percentage of the total for the whole city.

becomes lower, the time for saturation to occur decreases rapidly. *This stresses the importance of moisture from gas combustion as a factor in the formation of fog at the Edmonton airport.*

(b) The effect of wind on moisture supply

The determination of the effect of wind on the combustion moisture-supply necessitated the calculation of the ratio of the amount of moisture produced in oriented narrow channels to the total moisture produced over the whole city. These channels were 2 mi in width with the downwind end centred at the observing point at the airport. These ratios, expressed as percentages, are shown in the polar diagram of Fig. 4. The greatest amount of moisture (47 per cent) is added to the air in the channel to the south-south-east of the airport. The distribution of moisture sources around the airport explains in part why poorest visibilities occur with southerly winds and best with northerly winds.

The relative humidity of the air upon arrival over the airport after traversing a certain part of the city can now be calculated. The time required to reach saturation will vary from channel to channel, depending upon the concentration of gas consumption. If C is defined as the ratio of the consumption concentration over the whole city to the concentration in a channel, then the time required to reach saturation in a channel is,

$$t = Ct_1 \frac{D}{100}$$

when the relative humidity of the air entering the channel is zero.

The time for a mass of air to traverse the gas-consuming area in a channel can be expressed in terms of the area, and the speed of the wind.

Thus,

$$\tau = \frac{A}{2v}$$

where v = wind speed, A = gas-consuming area in a channel (of width 2 mi).

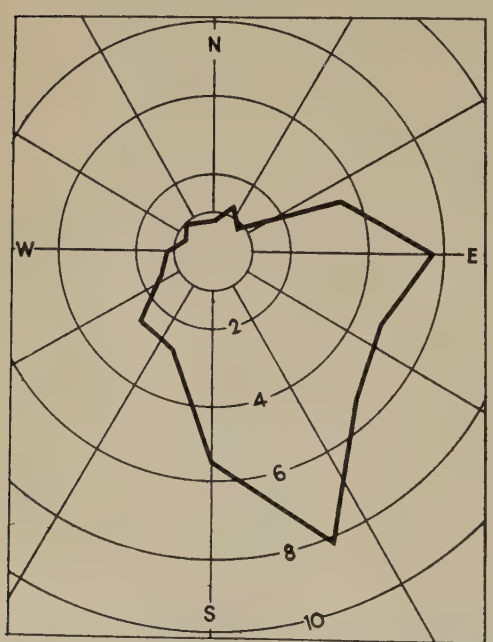

Figure 5. Factor A/C in various directions from airport.

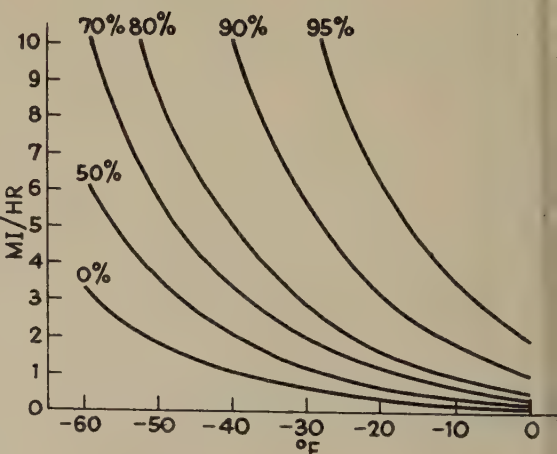

Figure 6. Temperature, wind speed, and initial relative humidity necessary to give saturation at the airport with a south wind and assuming a mixing depth of 200 ft.

Then the final relative humidity of the air upon reaching the airport is,

$$f = 100 \times \frac{\tau}{t} = \frac{10^4 \times A}{2vt_1 D C}$$

The polar diagram of Fig. 5 shows the variation of the factor A/C in mi^2, over the 16 channels extending from the airport towards the 16 points of the compass.

The depth, D, to which the moisture is mixed is uncertain, and may vary from one case to another. Using a value of 200 ft for D, which often has been confirmed by pilots flying over the city in cold weather, a graph was constructed showing the final relative humidity, f, for a channel from a specific direction for various wind speeds, and various air temperatures.

Since a S. or SSW. wind occurs most frequently at very low temperatures, curves for the final humidity of the air from the channel extending to the south were calculated. For this channel $A/C = 5{\cdot}6$, and using $D = 200$ the value of the final relative humidity is,

$$f = \frac{140}{vt_1}$$

Of more importance than the final relative humidity are the values of the wind speed, temperature, and initial relative humidity which will result in saturation of the air arriving at the airport. Setting $f = 100$, the initial relative humidity becomes,

$$U = 100 - \frac{140}{vt_1}$$

The critical values of U, v, and T necessary for a final relative humidity of 100 per cent are shown in Fig. 6. Then for supersaturation it is only necessary for :

(a) the temperature to be lower,
(b) the wind speed to be lighter, or
(c) the initial relative humidity to be higher.

Entering the graph of Fig. 6 at $-25°$F and at a wind speed of 2 or 3 mi/hr, it is found that a south wind will be saturated at the airport if the initial relative humidity is 80 to 85 per cent before the air reaches the city limits. There is no check on the initial relative humidity, but values within this range seem quite reasonable for returning polar continental air.

(c) *Other sources of moisture*

During cold weather there are two other possible sources of moisture which have not been considered : (a) the exhaust gases from automobiles, (b) the exhaust steam and combustion gases from locomotives.

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Secular changes of tropical rainfall regimes

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Summary

Secular changes are demonstrated by residual mass curves. It is shown that tropical rainfall decreased abruptly at the end of the 19th century. This was due largely to a contraction of the rainy belt and a shortening of the wet seasons. In the SE. Asian monsoon region the annual rainfall has varied less than in the rest of the tropics but changes occurred in the seasonal pattern. Similarities in the records suggest a physical connection between frontal or cyclonic activity in the temperate zone and the October-March monsoon rains of tropical Australia and SE. Asia.

A concluding section develops the argument that the evaporation-precipitation cycle is a partly unstable process and that small radiative changes could conceivably produce large and abrupt changes in the rainfall regime.

1. Introduction

Since C. E. P. Brooks (1919) opened the systematic study of climatic fluctuations from meteorological records, numerous investigations from many parts of the world have been published. Recently a very large and almost discontinuous change of the rainfall regime has been found to have occurred in SE. Australia (Kraus 1954). It was characterized by a reduction of spring and autumn rainfalls after 1893-4 which amounted to 50 per cent of the thirty-year mean for these seasons in the case of some stations. In a following paper it will be shown that this variation was paralleled on other temperate east coasts. On west coasts and in climatic regions of the Mediterranean type, the changes of the rainfall regime shown in the records are less pronounced and less abrupt. Fluctuations of a shorter period are more in evidence. Some of these, however, seem to have been of world-wide extent and it will be seen below that they appear also in some of the tropical records.

On the whole the rainfall regime of the tropics varied in the same way as that of the extra-tropical east-coast regions. The average annual volume of precipitation decreased rather suddenly at the turn of the century. This change, which occurred a few years later than at higher latitude, brought drought – or at least dry conditions – to tropical Australia, parts of the Pacific, Colombia, Mexico, NE. Brazil, the Caribbean, Africa, SW. Asia and Ceylon.

A local drought is usually comparatively short-lived. At least there do not seem to be any immediately discernible parallels oversea to the severe droughts which affected Queensland and northern New South Wales in 1918-9, 1922-3 and 1934-5. On the other hand, the widespread drought at the beginning of this century lasted – with some interruptions – at least fifteen years. Similarly, a tendency towards high rainfall affected most west coasts and winter-rainfall regions after 1915 for a period of several years. This apparent connection between the extent and the period of fluctuations might be of some practical value. If, for example, rainfall is found to be below normal all over the tropics, a prolonged dry period could be considered a strong possibility. This may open a way to forecast or anticipate fluctuations even if their mechanism is unknown. Though it is to be hoped that the investigation of climatic fluctuations will also reflect some light on the underlying causes in due course.

2. Method of representation

Climatic changes have been demonstrated usually by comparison between different thirty-year means or by running ten- and thirty-year averages. Both methods have serious disadvantages. They obscure the time when a trend changes and are misleading in the not infrequent case of the single exceptionally wet year affecting considerably the long-term mean. In general, the moving-average process applied to a random series gives rise to a series which is no longer random in character (Grant 1952). Runs of short length are suppressed both in number and amplitude whilst runs equal to the period of the running averages are increased in number.

In the present study, fluctuations were investigated by means of residual mass curves: graphs of cumulative, percentual deviations from the mean. This device is used commonly in hydrology to register an excess or deficit of stream flow over a prolonged period. Its use for the study of climatic changes was pioneered by engineers such as Keele (1915-6) and Barnes (1919).

The quantity plotted in the following graphs is given by,

$$y_n = 100 \sum_{l=1}^{n} \left(\frac{r_l}{\bar{r}} - 1 \right) - C,$$

where r_l is the rainfall for the lth year of the record and the mean rainfall $\bar{r}$ has been computed for the standard 60-year period 1881-1940 unless otherwise stated. The constant $C = 100 \sum_{1}^{n^*} \left(\frac{r_l}{\bar{r}} - 1 \right)$, the summation extending from the beginning of the record to n^*, which represents the year 1880, inclusive.

It is easily seen that y must be zero at the beginning of 1881 and at the end of 1940. The graph of y is rising during a period of years with average rainfall greater than $\bar{r}$ and vice versa. It will be valley-shaped, or concave upwards, if precipitation is increasing and convex when it is decreasing. The percentual deviation from $\bar{r}$ of the mean for any other arbitrary period, say the years n to $(n + m)$, is given simply by $(y_{n+m} - y_n)/m$.

Percentual deviations were used instead of simple deficits or excesses of volume so as to make records of widely differing regimes – registered either in cm or in. – comparable on the same scale.

Wherever some individual monthly records were missing from a long-term series, e.g. Barbados, July-December 1881, the rainfall was presumed to have been average.

3. The decrease of tropical rainfall

Fig. 1 shows the cumulative excess or deficit of rainfall for eight stations distributed over four continents and two oceans, and ranging in height from sea level to 8,250 ft.

All graphs show a rainfall regime above the 1881-1940 average during the nineteenth century, followed in most cases by an abrupt decrease. This pattern seems to have been world-wide, as far as can be judged from the records of every station, with a sufficiently long period of observation, published in *World Weather Records*. Exceptions occurred only in parts of SE. Asia, and these will be discussed later. The East African rainfall records start too late to be used in the present connection. However, residual mass curves of the Nile published by Keele (1915-6) show a very considerable reduction of the discharge after 1893.

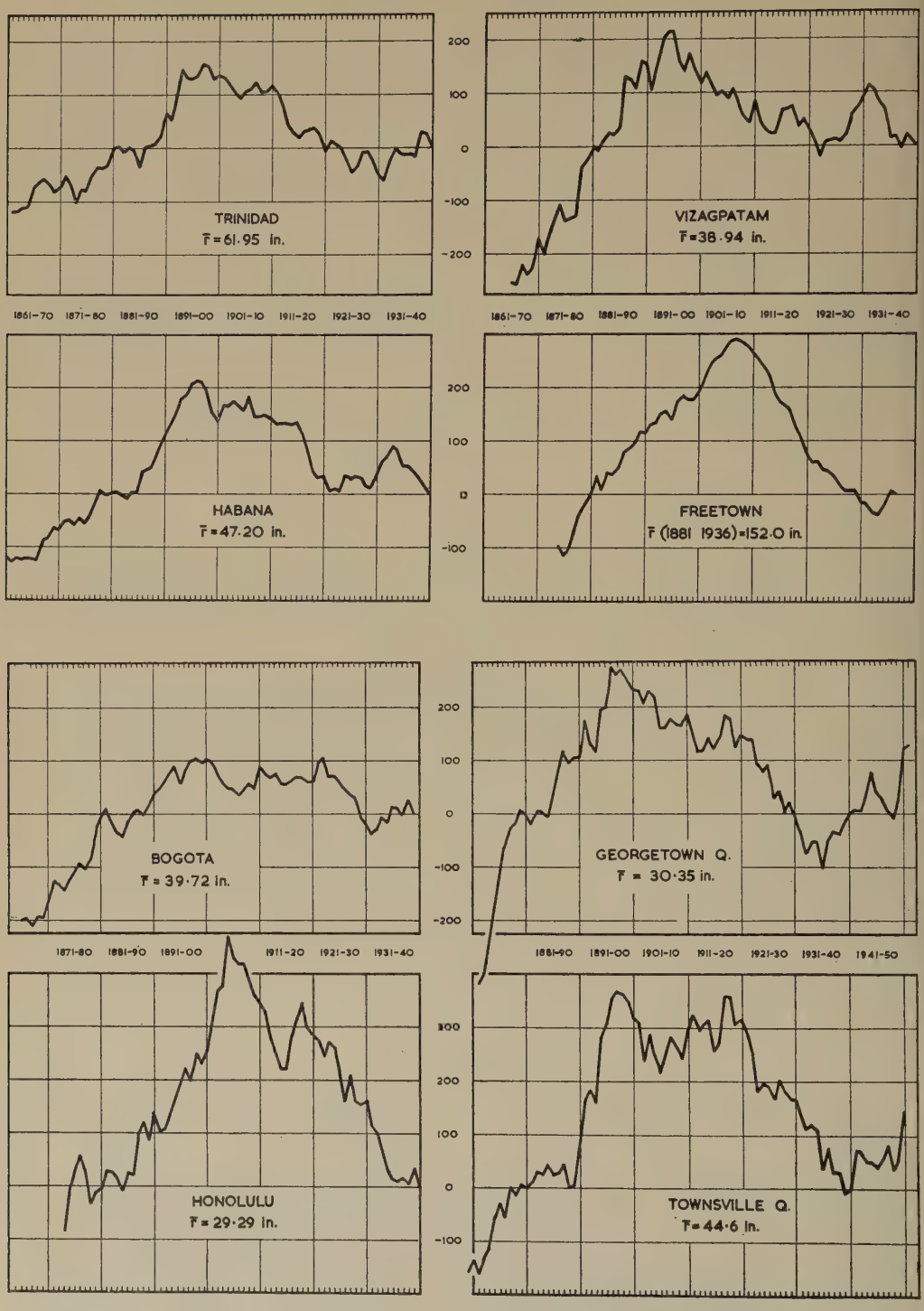

Figure 1. Cumulative percentual deviations from annual normal.

Table 1 shows the percentual deviations of precipitation from the 1881-1940 normal for eleven stations and two different twenty-five year periods.

TABLE 1. PERCENTUAL DEVIATIONS FROM 1881-1940 NORMAL OF MEAN ANNUAL RAINFALL FOR TWO SELECTED PERIODS

Station	Percentual deviation		
	1874-1898	1907-1931	Difference
Barbados (West Indies), 13°8'N 59°36'W	14	− 9	23
Bogota (Colombia), 4°36'N 74°5'W	10	− 3	13
Colombo (Ceylon), 6°54'N 79°53'E	4	− 8	12
Freetown (Sierra Leone), 8°29'N 13°9'W	11*	− 12	23
Georgetown (Queensland), 18°22'S 143°32'E	20	− 8	28
Habana (Cuba), 23°8'N 82°22'W	10	− 5	15
Honolulu (Hawaii), 21°19'N 151°52'W	13	− 12	25
Recife (Brazil), 8°4'S 34°53'W	42*	− 11	53
Townsville (Queensland), 19°15'S 146°48'E	17	− 4	21
Trinidad (West Indies), 10°40'N 61°31'W	10	− 7	17
Vizagpatam (India), 17°42'N 83°19'E	10	0	10

* Mean for 1875-1899; the early records from Recife, prior to 1881, appear to be unduly high.

Twenty-five years make rather a short period for the study of climatic variations. However, the differences given in Table 1 are all statistically significant. If the two means are considered samples from two populations, the standard error of the difference is 4·3 in the case of Habana for example. The actual difference is 15, that is 3·5 times its standard error, which is highly significant. In the case of Freetown the standard error of the difference is 4·1 and the actual difference 23 – more than five times as large. The real significance of Table 1 and of the figures lies of course in the internal consistency of records from all over the world.

General similarities apart, there are, however, some interesting differences between the records. The decline of the mean rainfall started in 1894 at Vizagpatam on the east coast of India. This corresponds exactly to the fluctuation pattern in SE. Australia (Kraus 1954). At most of the other stations, the decline started in 1897. However at Freetown, Honolulu and Colombo the decline did not begin until early in the 20th century. It has been surmised in the earlier paper that during the drought early this century, the upper westerlies over Australia moved comparatively close to the equator and it may be significant that Colombo, Honolulu and Freetown lie all on the western side of high mountains.

4. THE FLUCTUATION PERIOD

None of the records discussed in the preceding section began before 1859. From indirect evidence it seems probable, however, that the period of comparatively high tropical rainfalls began in the eighteen-twenties or earlier.

This conjecture is supported by analogy with stations on the east coast of the United States for which long-term records are available. In the present study, Fig. 4 below shows cumulative percentual deviations from the normal for the 'early rains' at Madras. These seem to have fluctuated in the same way as the annual rainfall in regions that are not affected by the Asian monsoon. It can be seen that they have been above the 1881-1940 normal through most of the nineteenth century. The most convincing evidence comes perhaps from Keele's residual mass curves for the Nile, which were referred to above, and which show an excess of discharge from 1835 to 1898.

At the other end of the record, it seems that the dry period which started at the beginning of this century may have ended, or at least have been interrupted temporarily, in the nineteen-thirties. At that time most of the curves in Fig. 1 show a tendency to flatten out or to rise. Records after 1940 were only available for Townsville and Georgetown – on opposite sides of the Australian Great Dividing Range. At both these stations the nineteen-forties and early fifties were much wetter than the preceding forty years.

It should be noted that the fluctuations described so far are not in phase with the retreat of glaciers, which appears to have started earlier than the decrease of the tropical rainfall regimes.

5. The arid fringe

In this and the following section, evidence will be produced to show that the decrease of the annual volume of rain was due largely to a narrowing of the rainfall belt and to a shortening of the wet season.

A contraction of the rainy season and expansion of the arid zones will have necessarily the largest effect upon stations on the arid fringe, where rainfall is always marginal and unreliable. For example, among the curves of Fig. 1, the largest amplitude is shown by Georgetown, which is close to the dry Australian interior and which is sometimes entirely without rain from April to October inclusive.

Even larger fluctuations occurred at the margin of the desert region which stretches from the Atlantic to India. For example, the residual mass curve for Aden in Fig. 2 shows that the 1881-1936 normal was exceeded by 67 per cent during the eleven years 1881-1891 and by 29 per cent over the twenty-five years 1881-1905. The deficit over the following twenty years 1906-1925 was 39 per cent.

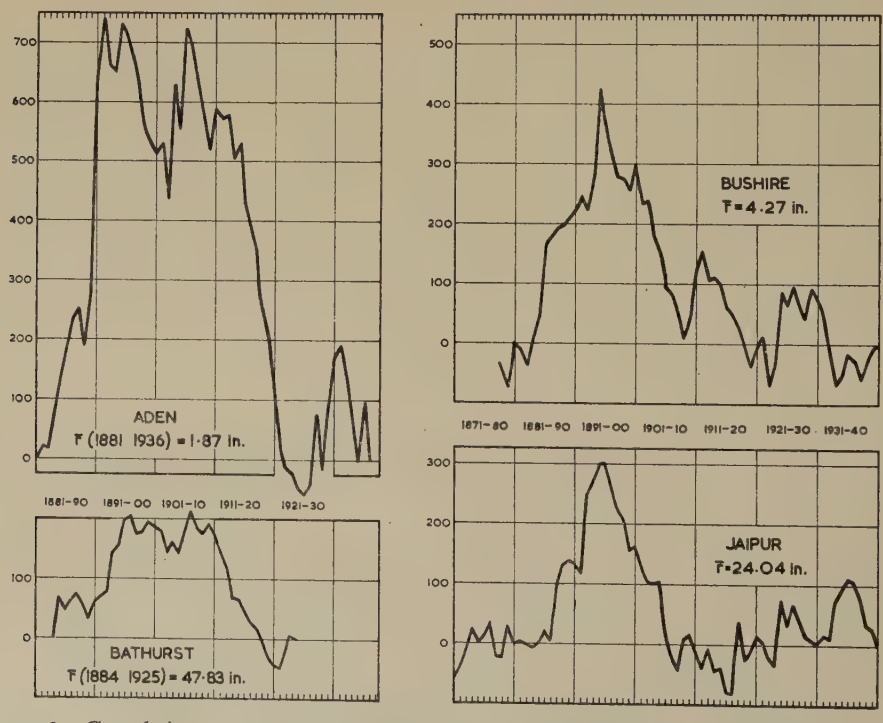

Figure 2. Cumulative percentual deviations from annual normal for stations on the arid fringe (Aden 13°N 45°E, Bathurst 13°N 17°W, Bushire 29°N 50°E, Jaipur 27°N 76°E).

Only 26 years of record are available for Bathurst on the western side of the Sahara. The rise and fall of the residual mass curve for this station is steeper than it is at Freetown further south along the same coast. Freetown is covered regularly by the inter-tropical convergence zone for part of the year and its normal annual rainfall is more than three times heavier than that of Bathurst. It appears that, after the turn of the century, the equatorial rains failed more frequently than before to penetrate as far north as Bathurst.

Something similar may have happened at the eastern end of the desert belt. Most of the Indian monsoon region shows no significant secular change of the rainfall (Pramanik and Jagannathan 1953), but Jaipur – at the fringe of the monsoon region, close to the Thar desert – exhibits an appreciable variation which is also represented in Fig. 2.

Both Bathurst and Jaipur are typical summer-rainfall stations; in fact the Bathurst mean rainfall for the months January-April is given as zero. Bushire on the south coast of Iran, on the other hand, receives its rain from westerly ' Mediterranean ' disturbances. One of the most extreme winter-rainfall stations, it experienced, in 65 years, only one single rainfall during the four summer months June-September. The similarity of the rainfall variation in Bushire and Jaipur suggests that the desert has tended to expand both ways – towards the equator and towards the temperate zone.

6. The shortening of the wet season

The inter-tropical convergence zone moves north and south with the sun. A narrowing of this zone must shorten the wet season at any particular fixed location.

The change towards a longer dry season and shorter wet season is demonstrated for Freetown in the following table.

TABLE 2. Average monthly rainfalls at Freetown (in.)

	J	F	M	A	M	J	J	A	S	O	N	D
1875-1896	0·60	0·50	0·92	5·80	15·17	25·17	40·32	45·09	35·68	16·62	6·54	2·05
1907-1931	0·16	0·12	0·81	2·46	8·49	16·92	33·12	31·04	23·08	10·20	4·66	0·89
Ratio	0·27	0·24	0·88	0·43	0·56	0·67	0·82	0·69	0·65	0·61	0·71	0·44

The table shows how rainfall tended to become more concentrated in time. The relative decrease of the early rains (May and June) and of the late rains (September and October) is appreciably larger than that of the wettest months (July and August) – though this does not apply to the absolute volume deficit.

After 1896 the dry season became more arid as evidenced by Table 3.

TABLE 3. Number of January records for Freetown with a rainfall of more than 0·1 in.

	1876-1895	1896-1915	1916-1935
Januarys with rainfall above 0·1 in.	10	4	5

Both the early and late rains decreased after 1897 and the continuous rise of the Freetown curve in Fig. 1 up to 1906 is due to a series of particularly heavy July and August rains. The same applies to Bathurst further up the coast where it caused the two distinct peaks on the curve in Fig. 2.

Conditions on the other side of the Atlantic are illustrated by the two curves for Barbados in Fig. 3. The amplitude of the fluctuation was much larger for the dry season January-May than for the wet part of the year, suggesting once more increasing aridity and a shortening of the rainy period.

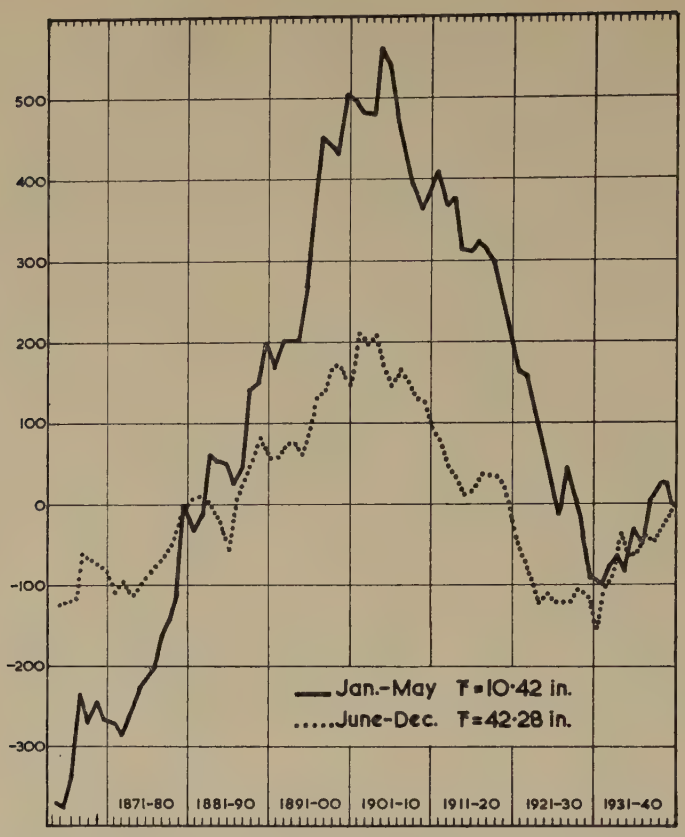

Figure 3. Cumulative percentual deviations from seasonal normals : Barbados (West Indies).

7. The changing seasons of SE. India

Although a diagram published by Sir Charles Normand (1953) suggests a tendency for India to suffer from droughts after 1898 together with the rest of the tropics, the recorded annual rainfall amounts – except for the semi-arid region of the north-west and the middle east coast around Vizagpatam – fail to exhibit the common world-wide pattern of climatic change. What fluctuations there are tend to be insignificant in most cases. This has been deduced from numerous residual mass curves that were prepared during the present study and it is confirmed by Pramanik and Jagannathan's very thorough analysis (1953). However, the apparent confusion and lack of trends in the Indian records may well be deceptive and it seems possible that they can be resolved in terms of the general pattern of tropical change and other disturbing or opposing factors. Some aspects of this problem are discussed below, though a complete analysis, which would be very laborious, was considered to be beyond the scope of the present paper.

Systematic rainfall measurements seem to have been carried out in India first in Madras, for which an unbroken record back to 1813 is available. The mean annual rainfall during this period never fluctuated significantly. In fact mean precipitation for the years 1813-1880 was 49·87 – exactly the same as for the years 1881-1940. The picture changes completely when the annual records are broken down. Fig. 4 shows residual mass curves for the early-rain months May-July and for the main rainy period October-December which in Madras is due to the NE. monsoon.

It will be seen that the curve for the early rains shows excess rainfall through most of the 19th century and a change of gradient about 1893. For the period of simultaneous

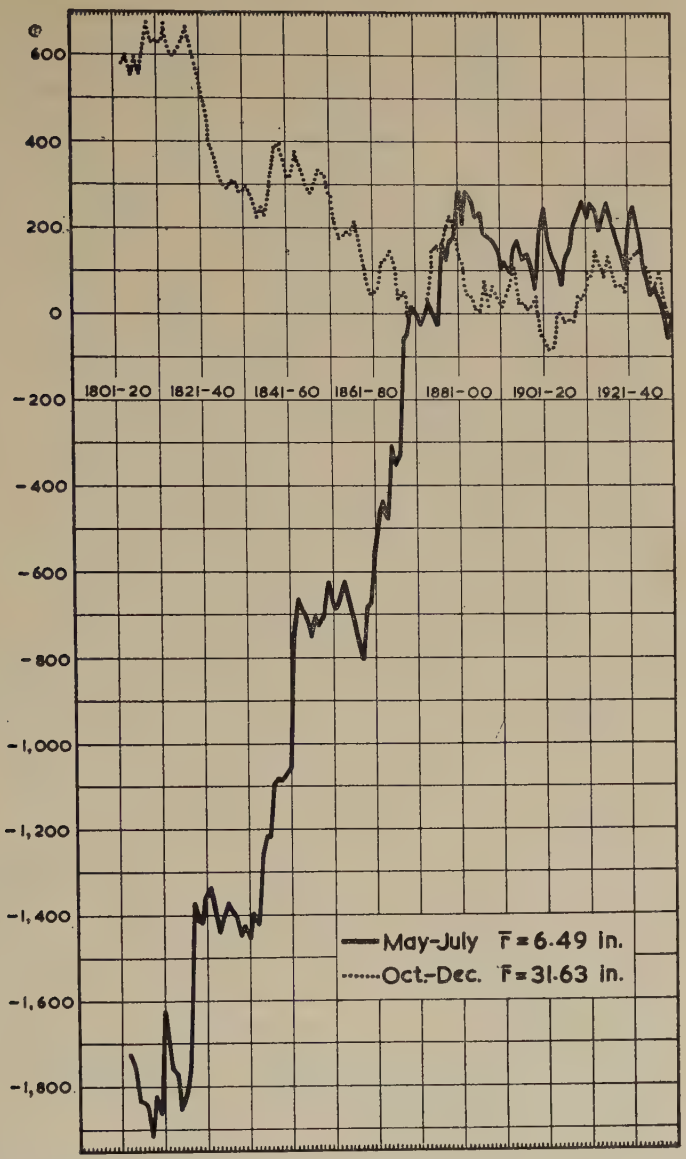

Figure 4. Cumulative percentual deviations from seasonal normals : Madras (India).

records, the early rains of Madras varied therefore in a similar fashion to the annual rainfall in most other tropical and east-coast regions. The October-December rains, on the other hand, were below the 1881-1940 normal in the 19th century and they have shown a tendency to increase.

Two considerations appear to be of particular interest. Firstly the change in the mean rainfall is much larger in May-July during the SW. monsoon, Madras then being to leeward, than in November and December when the wind blows onshore. The same happened in the temperate westerly belt, where east-coast stations tend to exhibit much larger and more consistent secular fluctuations of the rainfall regime than west-coast stations. Secondly the two curves in Fig. 4 show an opposing trend. This caused the absence of any trend in the yearly values. It is not impossible that the same influence which caused the SW. monsoon rain to decrease, may have caused the NE. monsoon

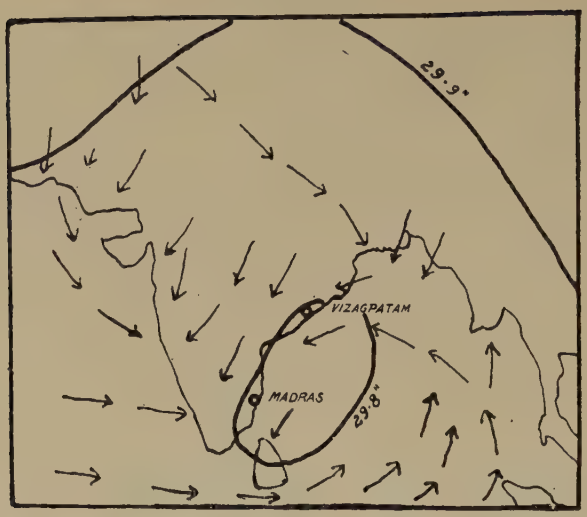

Figure 5. Winds and pressure distribution over the Indian region in October (after Philips' Atlas).

rain to increase. A change in the mean strength of the upper westerlies or in their mean distance from the equator could well produce such a result.

Further north along the Indian east coast, Masulipatam and Vizagpatam are among the few Indian stations to provide records which show the same secular variation of the annual rainfall as is observed in other parts of the tropics (see Fig. 1). The cause may become apparent from a study of the monthly values.

TABLE 4. MEAN MONTHLY RAINFALL AT VIZAGPATAM (IN.)

	J	F	M	A	M	J	J	A	S	O	N	D
1866-1895	0·28	0·18	0·36	0·68	2·80	5·29	4·69	6·16	7·94	11·45	4·73	0·80*
1896-1930	0·47	0·84	0·42	0·83	1·83	3·62	4·46	5·33	6·93	6·60	4·12	0·52
Difference	0·19	0·66	0·06	0·15	− 0·97	− 1·67	− 0·23	− 0·83	− 1·01	− 4·85	− 0·61	− 0·28
Ratio	1·68	4·66	1·17	1·22	0·65	0·68	0·95	0·86	0·87	0·58	0·87	0·65

* December 1878 omitted.

Table 4 shows an increase of the rainfall for the months January-April. The decrease for the May-July season corresponds to the same change in Madras. A small decrease in the months of August and September is also a feature of conditions further south. However, this similarity breaks down completely in October. Whilst the October rainfall in Madras increased, it decreased sharply in Vizagpatam, where this is the wettest month of the year.

The apparent discrepancy between Madras or such places as Trincomalee on the east coast of Ceylon on the one hand, and Vizagpatam or Masulipatam on the other hand can perhaps be explained in terms of a low which forms frequently in October off the Madras coast, where it appears even on mean isobaric charts. It will be seen from Fig. 5 that this is associated with onshore winds in the Vizagpatam region, and with winds parallel to the coast or offshore further south. If analogy with the east coast of Australia is any guide, these east-coast autumn lows become less frequent after 1894 and if this is true the same must have happened to the rain-bearing, onshore winds of the Vizagpatam region.

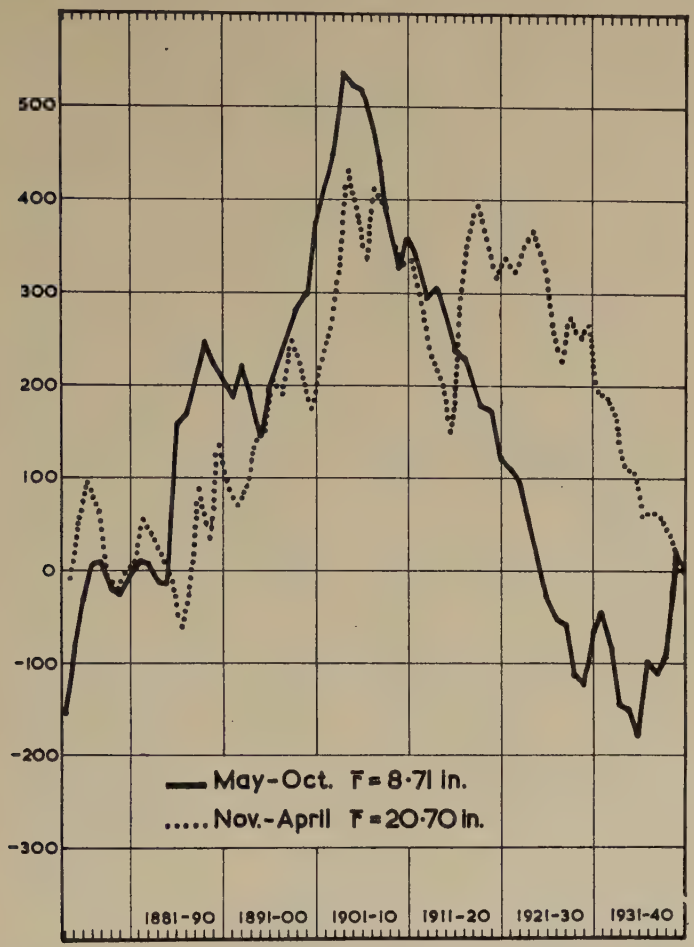

Figure 6. Cumulative percentual deviations from seasonal normals : Honolulu (Hawaii).

Confirmation of this hypothesis could be obtained from a systematic investigation of a long series of synoptic charts, and such an approach might well succeed in resolving other confusing features of the Indian rainfall records.

8. Disturbances of the middle latitudes and the Australian-Asiatic monsoon system

The curve for October-December in Fig. 4 indicates a marked excess of rain in Madras, for the NE. monsoon seasons 1912-1922. This excess was not limited to Madras. For example, on the curves for Honolulu, Georgetown and Townsville in Fig. 1, it led to the peculiar 'hump' or secondary maximum of the cumulative percentual deviation in 1917-8. The same hump – though less pronounced – can be seen in the Barbados curves. In Bogota, the culminating point is reached somewhat later in 1922. It appears that some of the excess tropical rainfall which led to this hump and the subsequent decrease was associated with changes of perturbation activity in the westerlies.

Most of the years 1914-18 tended to have above average rainfall at many stations in temperate regions with a westerly rainfall-regime and a winter maximum, such as San Francisco, Lisbon, Funchal and Valencia. That the same happened in England may be confirmed from the curves published by Barnes (1919). The case of Honolulu is

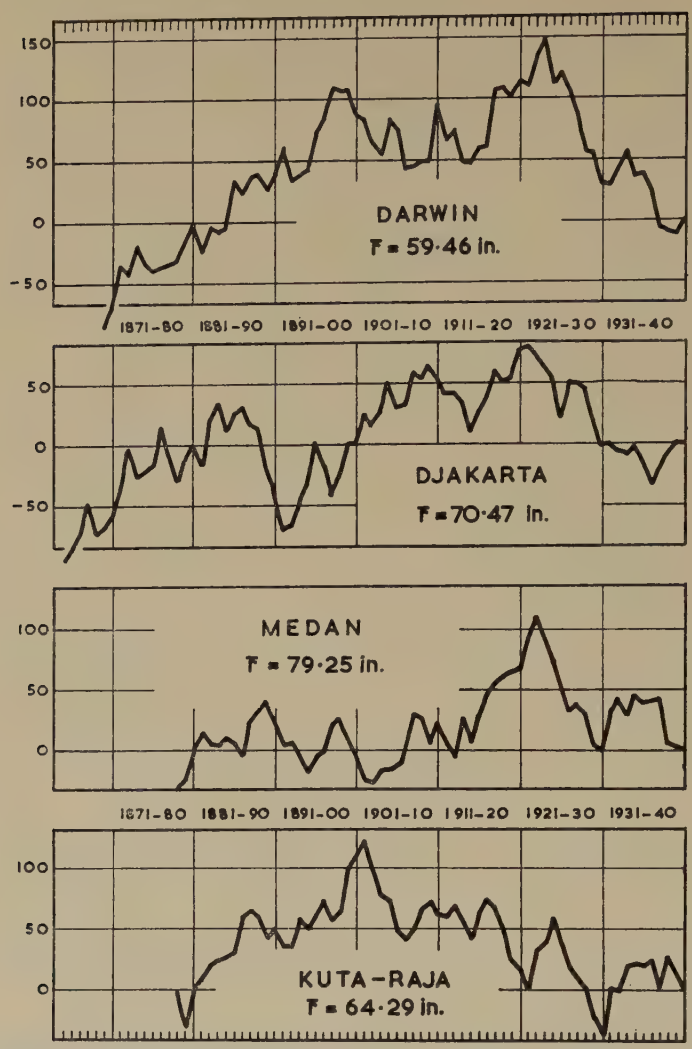

Figure 7. Cumulative percentual deviations from annual normal for stations in the Australian-Asiatic monsoon region (Darwin 12°S 131°E, Djakarta 6°S 107°E, Medan 4°N 99°E, Kuta-Raja 5°N 95°E).

particularly instructive. The winter rainfall there is predominantly of a frontal nature (Riehl 1954). This is less the case in summer. Fig. 6, which gives cumulative percentual deviations for winter and summer rainfall separately, shows that the excess rainfall there in 1915-7 and in 1921-4 was limited to winter and hence probably to frontal activity.

Fig. 7 traces this fluctuation across the Pacific and Indian Oceans (the ratio of the vertical to the horizontal scale is 1·5 times larger on this figure than on the others). The stations shown receive most or all of their rain from the Asiatic monsoon during the northern winter. In Darwin, 12° south of the equator, the normal pattern of change, with excess rainfall up to 1897 and a following deficit is still clearly evident. The subsequent period of excess precipitation from 1914 to 1923 is, however, even more pronounced. At Batavia, 1500 mi to the west of Darwin, the peak in the late eighteen-nineties has completely disappeared. The curve culminates in 1921, though the variations are complex and very small. The same applies to Medan on the north-east coast of Sumatra. It is of interest that Kuta-Raja – only 200 mi from Medan, but on the west coast and sheltered from the NE. monsoon – has the normal pattern of change with a peak of its residual

nass curve in 1902. However, further west, the Indian and Ceylonese stations which eceive their rain with the NE. monsoon show again a residual-mass curve peak in 1922. The outlined similarities in the fluctuations of the rainfall regime from the Hawaiian winter lown to North Australia, Indonesia and Ceylon suggest some fairly direct physical onnection between the northern westerlies and the monsoon, which again might well e a rewarding subject for more detailed synoptic study.

9. The instability of the evaporation-precipitation cycle as a possible cause of discontinuous climatic change

Although the common pattern of climatic change is by no means absent from the E. Asian monsoon region, the change in annual rainfall volumes has been comparatively mall during the period from which records are available. However, this region occupies nly a fraction, approximately one tenth, of the tropical belt. Over most of the rest, the ainfall decreased appreciably and abruptly about the beginning of this century. In articular that seems to have been the case over both the Pacific and the Atlantic Oceans, s far as can be judged from island and coastal records.

From Fig. 1 or from Table 1 it may be estimated that the overall decrease was erhaps of the order of 20 per cent. This was not compensated by any excess rainfall n temperate regions. To the contrary, on the east coasts of North America and of Australia the rainfall decreased as substantially as in the tropics, whilst in most other egions the changes were small and variable. The reduction of tropical rainfall must ave been associated therefore with a considerable weakening of the evaporation-preci- itation cycle.

Almost all precipitation water originates from the oceans, where evaporation E can e represented as a function of the wind speed W_a at a height a, and of the difference etween the vapour pressure at the sea surface e_w and in the air e_a (Jacobs 1951),

$$E = K\,(e_w - e_a)\,W_a$$

The value of the ' evaporation factor ' K depends on the character of the sea surface. t is about 3·5 times as large when the sea surface is hydrodynamically rough as when t is hydrodynamically smooth. Transition from smooth to rough conditions occurs at a windspeed of about 650 cm sec^{-1} measured at a height of 6 m. On the basis of these considerations, Jacobs deduced the following equation for the mean evaporation,

$$\bar{E} = K' \{n\,\overline{[(e_w - e_a)\,W_a]}_{W_a<650} + 3{\cdot}5\,m\,\overline{[(e_w - e_a)\,W_a]}_{W>650}\},$$

where n and m represent the fraction of time with windspeed less or greater than 650 cm sec^{-1}. Neither the threshold velocity of 650 cm sec^{-1}, nor the numerical value of the factor assumed here to be 3·5, are generally accepted. The following argument remains, however, valid even if this factor is unity, which is not likely.

The heat necessary to evaporate the sea water is supplied almost exclusively from the ocean and not from the atmosphere. The observed decrease of mean precipitation and evaporation could hence be due conceivably to a cooling of the sea surface. However, the observed decrease of precipitation by perhaps twenty per cent or more, would require a lowering of the tropical sea-surface temperature by at least 4°F, which is of the same order as the difference between the ice age and the present period (Brooks 1951) and which certainly did not occur between the 19th and 20th century. In fact, if there was any change at all, it was probably a small variation in the opposite direction.

It also seems probable that the value of $(e_w - e_a)$ remains more or less constant, when averaged over the whole surface of the tropical oceans and over a sufficiently long period of time. The decrease in evaporation is hence probably connected with a decrease

of mean windspeeds. It could be strongly affected by change of the ratio n/m in particular. The charts published by Riehl (1954) indicate that the mean wind over large areas of the sub-tropics is marginal in the sense that it can be either above or below 650 cm sec^{-1}. A comparatively small shrinking of the surface area which is hydrodynamically rough could produce easily a large decrease of mean evaporation.

Also, as discussed earlier, both the amplitude and the shape of the rainfall fluctuations seem to have been influenced by the orographic configuration. This again suggests a close connection between the observed changes in the rainfall regime and the average windspeed and pattern.

It would be of great interest to know why the windspeed and pattern changed at the end of last century. The circulation of the tropical and sub-tropical latitudes is driven by a system of heat- and cold-sources which includes conductive heating in the trade-wind regions, mainly in the eastern part of the sub-tropical oceans, and cooling by a net loss of long-wavelength radiation from the upper troposphere. The amount of heat which the air receives by direct conduction from the oceans is, however, very much smaller than the amount gained as latent heat by evaporation. This latent heat is converted into sensible heat, and so becomes dynamically effective, mostly in the narrow equatorial region where the bulk of the water is condensed and precipitated. A reduction of the equatorial rainfall by something like 20 per cent reduces this important heat source of the circulation by an equal factor.

It follows that the evaporation-precipitation cycle is partly at least an unstable process – particularly in the tropics. A small reduction of precipitation will reduce the amount of heat made available to the atmosphere. This in turn will reduce the kinetic energy of the circulation and so the mean speed of the trades and, more important perhaps, the sea-surface area which is hydrodynamically rough. The resulting decrease of evaporation should further reduce precipitation and so on.

The initial disturbing impulse might come from a change in the stability of the upper troposphere, or from a change in the intensity and mean originating height of outgoing long-wavelength radiation, or from a change in the mean height of the tropopause. All these could be affected in turn by a change of incoming radiation. From the argument of the last paragraph it would follow that, even if the initial change is very small and continuous, it could produce conceivably the large and rather abrupt changes of the rainfall regime that have been described in this paper.

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World Weather Records		Smithsonian Miscellaneous Collections.
	1927	**79.**
	1934	**90.**
	1947	**108.**

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Electrostatic field-changes due to lightning discharges

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Summary

Extensive observations are described of the changes of electrical field produced by lightning discharges at distances of less than 100 km; in this range electrostatic effects are dominant. The results are analysed in detail.

A long series of capillary-electrometer records obtained with auxiliary amplifying equipment give statistical information upon the magnitude-distance and sign-distance relationships for the total electrostatic field-change due to a lightning flash. From oscillographic observations it is shown that most of this change in field occurs comparatively slowly. It has proved possible to separate field-changes into distinct groups; these are considered in turn and values given for such quantities as the electric moment and the times involved in each stage, etc.

Finally, meteorological influences and the relative frequency of occurrence of cloud and ground discharges are investigated.

1. Introduction

If q and h are the charge and height of an element of volume, the electric moment M of a thundercloud may be written as $2\Sigma qh$, where the summation covers all the charges associated with the cloud. Provided the distance D to an observing station is large compared with the dimensions and height of the thundercloud and that ionospheric effects may be neglected, the vertical potential gradient or field E_t at the earth's surface is given, at any instant t, by

$$E_t = \frac{M}{D^3} + \frac{1}{cD^2}\frac{dM}{dt} + \frac{1}{c^2 D}\frac{d^2M}{dt^2} \quad . \quad . \quad . \quad (1)$$

In this familiar equation, c is the velocity of light, and the three terms, usually called the ' electrostatic (E_s),' ' induction (E_I),' and ' radiation (E_R),' fields, respectively, have the retarded values existing at times $(t - D/c)$. When a lightning discharge, which may be considered as moving a charge Q through a height H, occurs, the initial and final values of E_I and E_R are zero, and the net or semi-permanent change in field produced by the flash is given by the electrostatic component, $E_s = M/D^3$, alone.

Wilson, (1916 1920) pointed out that the greater D, the more valid are the approximations involved in Eq. (1) and the more closely does the quantity $(E_s D^3)$, which he termed the ' uncorrected moment ' of the discharge, approach to the true value $2QH$ of the moment involved. Working at Cambridge and using a capillary electrometer, Wilson made the first reliable observations of field-changes and was able to estimate the charges involved in thunderclouds. Similar measurements were continued by Wormell (1939). During the same period, cathode-ray oscillograph work by Appleton and Chapman (1937) and others, resulted in a more detailed knowledge of the form of the electrostatic field-change associated with an individual flash.

One feature of the capillary-electrometer observations, namely the frequent occurrence of field-changes of considerable duration, ($\geqslant 0{\cdot}25$ sec), during which the change of field took place in a slow steady manner, remained unconfirmed by the oscillographic records. Cathode-ray equipment capable of responding faithfully to very low frequencies was therefore constructed at Cambridge, and the present paper deals in detail with the results obtained by this technique. In addition, records were obtained simultaneously, by

means of a capillary electrometer with a simple auxiliary amplifier; these observations, besides supplying valuable statistical data and maintaining the continuity of field-change studies upon the same site in Cambridge, permitted a direct comparison of the results obtained by the two techniques.

Investigations in 1947 soon revealed that, for the great majority of lightning flashes, most of the associated electrostatic field-change occurred comparatively slowly, and that previous investigators had underestimated the size and duration of the slow sections by factors of up to ten or more. Field-changes due to flashes within clouds were found to be entirely slow in character, while those caused by discharges to earth contained large slow stages before, after, and between, the more rapid elements associated with return-strokes. A brief preliminary description of the work, emphasizing the importance of the slow effects, was given in 1948 (Wormell and Pierce 1948), and further summaries have since been published (Wormell 1953a; Wormell 1953b; Pierce and Wormell 1953), but the present paper represents the first complete description of these results. The importance of the slow effects in electrostatic field-changes has also been demonstrated by Malan and Schonland (1951, 1953); they have been particularly concerned with the stages between the return strokes of discharges to earth.

2. Apparatus and recording technique

The open recording site and Wilson-sphere aerial, described by Wormell (1939), were used in these observations, and after applying small corrections (< 7 per cent) for the influence of neighbouring objects, the final measurements represent, within the limits of the frequency response of the whole equipment, the true magnitude and form of the variations in E.

The capillary electrometer and oscillograph amplifiers were both fed through electrometer valves from the same common aerial circuit, in which a capacitance, variable in steps, each with an associated grid-leak maintaining the time-constant at 10 sec, provided the sensitivity control; at maximum sensitivity the capacitance was 250 $\mu\mu$f and the leak 4×10^{10} Ω.

Essentially, the capillary electrometer and recording arrangements employed, were as used by Wormell (1939); the additional amplifier preceding the electrometer was of simple battery-coupled design, the frequency response of the entire system being level from below 1 c/s to above 20 c/s. On the final record the time resolution was 0·05 sec; since, for a lightning discharge, the initial and final values of E_I and E_R are zero within a time less than this, the record only represents changes in E_s. With maximum sensitivity, field-changes, of 0·02 v/m could just be detected. In practice, sensitivity is limited by fields due to power lines and by the incessant fluctuations of the normal potential gradient; these latter are sometimes associated with local turbulence, and often have quasi-frequencies approaching 2 c/s or more with amplitudes, even in fine weather, of up to 10 v/m.

In the oscillograph equipment, the amplifier was of conventional resistance-capacity coupled design, the output representing the changes in E with a level frequency response varying from 1 c/s to 4 kc/s. A continuous recording method, similar to that of Chapman (1939), was used, a time-base (period 200 msec), deflecting horizontally on the cathode-ray tube, being combined with a camera in which the film moved vertically. Vertical deflections on the tube were due to the field E. Under undisturbed conditions, the record showed a series of slanting traces corresponding to the successive traverses of the time-base; a rapid field-change gave an almost instantaneous vertical jump while a slower change in field merely altered the natural inclination of the trace, and in measurement

correction for the motion of the film sometimes had to be made. Times of 1 msec could be resolved. In recording, the brilliancy of the tube was maintained at a constant level, at which the very rapid variations in E_R and E_I associated with a fast electrostatic change, although visible on the screen, did not affect the film.

Some supplementary investigations of rapid variations in field were made with an automatic brilliancy control. Records were also obtained with auxiliary cathode-ray equipment using a 'triggered-raster' technique of high time-resolution (Caton and Pierce 1952).

3. Capillary electrometer results

The use of a supplementary amplifier in conjunction with the capillary electrometer enabled storms, considerably more distant than in previous work (Wilson 1920, Wormell 1939), to be investigated. Variations in the magnitude and sign of field-changes with distance, D, were studied for a range from 0 to over 100 km. The observations to be discussed were made during the summers of 1939, 1946 and 1947.

3.1 The magnitude-distance relationship

Most of the present results were obtained for storms with $D > 40$ km; at these distances the uncorrected moment $E_s D^3$ approximates closely to the true value of $2QH$. During any recording period, the positions of the active storms were derived from data supplied by two sources, namely, the 'Sferics' network of the British Meteorological Office (Ockenden 1947), and the British Thunderstorm Census Organization (Bower 1950). The information from these two bodies was usually adequate and comprehensive, although, on rare occasions, brief outbreaks of thundery activity were undetected by either system.

For each electrometer record (duration 10 min), the three largest positive* P_1, P_2 and P_3, and the three largest negative N_1, N_2 and N_3, field-changes, satisfying the relations

$$P_1 \geqslant P_2 \geqslant P_3 \qquad N_1 \geqslant N_2 \geqslant N_3 \quad . \quad . \quad . \quad \text{(2a)}$$

$$P_1 \not> 2P_3 \qquad N_1 \not> 2N_3 \quad . \quad . \quad . \quad \text{(2b)}$$

were measured, and the means $P = \frac{1}{3}(P_1 + P_2 + P_3)$ and $N = \frac{1}{3}(N_1 + N_2 + N_3)$ determined. Complex field-changes containing sections of both signs were regarded, for the purposes of relations (2a) and (2b), as consisting of a positive and a negative field-change equal to the sum of the individual positive and negative parts, respectively. P and N were then accepted as the average magnitudes of the field-changes due to the nearest active storm (distance D).

Most records included one or two field-changes considerably exceeding all the others in magnitude; these were usually negative and were probably associated with exceptional discharges of great length. Discrimination against these very large field-changes was introduced by means of the criteria (2b) in order that P and N should represent the typical rather than the maximum size of field-changes from a given distance.

In Figs. 1 and 2, P and N are plotted against D, each dot representing a 10-min record. The straight line is the line $ED^3 = 10^6$ (v/m) $\times$ km^3, which in each case is nearly the median line through the points. It corresponds to a value of 110 coulomb $\times$ km for $2QH$; this figure is in excellent agreement with the observations of Wormell (1939) for his maximum range in distance. For both positive and negative field-changes, the

* Sign convention : in a positive field the potential increases with height; a positive field-change produces an augmented positive field.

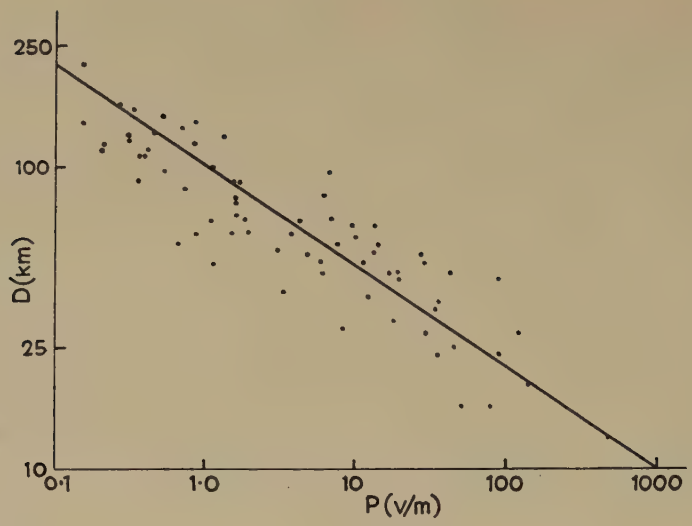

Figure 1. Mean field-change magnitude against distance for positive field-changes.

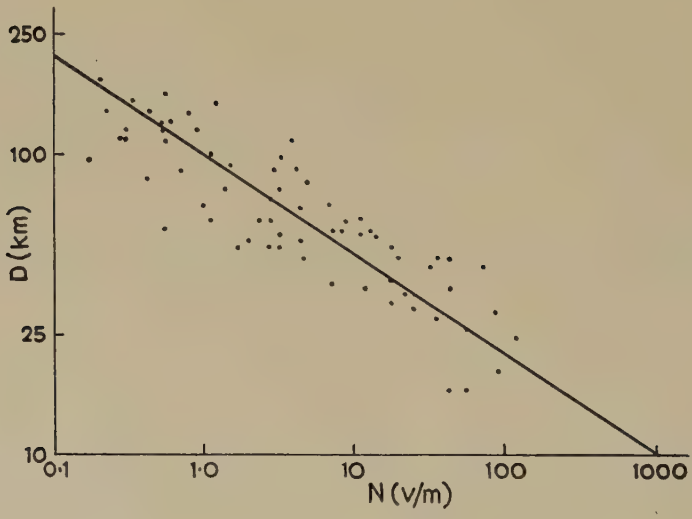

Figure 2. Mean field-change magnitude against distance for negative field-changes.

present results give an average value of 150 C km for $2QH$. This is rather less than Wormell's figure; the discrepancy is almost certainly due to the rejection of very large field-changes by the criterion (2b).

It has been established that the typical magnitude of field-changes, for distances 25 to 250 km from the discharge, obeys the expected inverse cube relation. The median magnitude, for either sign, corresponds to a change of electric moment of 110 C km, i.e., to a field-change of 1 v/m at 100 km; later evidence (§ 4.5) suggests that this figure may be an overestimate of 20 per cent for positive field-changes and of 35 per cent for negative changes in field. Individual field-changes from a storm at any given distance may vary in size by a factor of twenty to one or more; the spread in magnitude is particularly marked for negative field-changes.

TABLE 1. SIGN-MAGNITUDE RELATIONSHIP FOR SIMPLE FIELD-CHANGES

Magnitude of field-change (v/m)	Approximate corresponding range in distance (km)	Year and source of observations: 1920-4 Wilson			1926-36 Wormell (1939)			1939 present records			1946 present records			1947 present records			1949 present records			1920-49 all sources		
		N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-	N_+	N_-	N_+/N_-
> 10,000	< 4	22	0	–	38	15	2·5	8	0	–	0	0	–	0	0	–	0	0	–	68	15	4·5
5 000–10,000	4–6	41	3	14	76	23	3·3	5	1	5·0	0	0	–	0	1	–	0	0	–	122	28	4·4
500–5,000	6–12	159	40	4·0	640	276	2·3	92	18	5·1	26	2	13	3	6	0·50	3	1	3·0	923	343	2·7
100–500	12–20	117	99	1·2	452	248	1·8	100	59	1·7	23	19	1·2	31	15	2·1	2	0	–	725	440	1·6
10–100	20–50	249	295	0·84	1192	944	1·3	663	341	1·9	119	127	0·94	155	184	0·84	5	2	2·5	2383	1893	1·3
1–10	50–100	0	0	–	279	319	0·88	1089	583	1·9	281	344	0·82	333	550	0·61	44	19	2·3	2026	1815	1·1
0·5–1	100–130	0	0	–	0	0	–	310	168	1·8	121	133	0·91	74	151	0·49	20	13	1·5	525	465	1·1
0·1–0·5	130–200	0	0	–	0	0	–	319	186	1·7	396	389	1·0	51	91	0·56	9	19	0·47	775	685	1·1
< 0·1	> 200	0	0	–	0	0	–	99	59	1·7	124	124	1·0	0	1	–	1	0	–	224	184	1·2

3.2. *The prevailing sign of field-changes*

It is well known that near a storm most field-changes are positive, while as the distance of the activity increases negative field-changes become more frequent. The present observations, since they cover a wide range in distance, enable this effect to be investigated more closely than before. When several storms are active simultaneously, as is usually the case, it is probably legitimate to associate the larger field-changes with the nearest storm-centre (§ 3.1), but this process cannot be extended to the more distant storms. Consequently the classification of field-changes by sign was made according to magnitude rather than distance. Figs. 1 and 2 indicate a systematic variation between distance and magnitude with almost the same moments involved for both positive and negative field-changes (see, however, § 4.5), and since Wormell (1939) when classifying field-changes according to sign obtained identical results from both distance and magnitude criteria, the course adopted is considered well-founded.

Table 1 represents the results obtained at Cambridge during the years indicated; the numbers of all positive (N_+) and negative (N_-) simple, (i.e., not complex), field-changes recorded, are given for several ranges of magnitude, the approximate corresponding ranges in distance being also shown. The observations in 1949 were made with the oscillograph; otherwise, the information was derived from capillary electrometer records.

Two important points are evident from Table 1. (1) For any particular year and for magnitudes less than about 100 v/m, the ratio N_+/N_- is constant. This constant value may differ from year to year, but there is no significant change with magnitude between 100 and 0·1 v/m. (2) Above 100 v/m positive field-changes become increasingly predominant as the magnitude of the field-change rises.

The interpretation of these facts is clear. By § 3.1, a field-change of 100 v/m corresponds to a distance of about 20 km; the constancy of N_+/N_- below 100 v/m therefore implies that discharges, producing a reversal in the sign of the associated field-change beyond 20 km, do not occur.

An appreciable proportion of the discharges observed within a year often occurs during one thunderstorm, for instance in 1947 electrometer records were obtained on eight days but the storm of the 28th July was responsible for 55 per cent of the field-changes recorded in the year. Great differences in the proportions of positive and negative field-changes were found between individual storms; alterations in N_+/N_- also took place during the course of any one storm, although these were not so marked and were not systematic in character. The changes from year to year in N_+/N_-, for field-changes < 100 v/m, are therefore not to be regarded as characteristic of the year but rather as representing differences between particular storms.

3.3. *Complex field-changes*

Following Wormell (1939), with whose observations the present results where comparable are in good agreement, the magnitude and sign of a complex field-change may be defined as those of the larger component obtained when the field-change is reduced to two components, positive and negative, by adding the individual positive and negative sections. With these definitions, the present results showed that the proportion of complex field-changes observed decreased with magnitude; in the ranges 5,000-500, 500-100 and 100-1 v/m, 43 per cent, 35 per cent and 20 per cent respectively, of all field-changes recorded, were complex.

A sign-magnitude relationship similar to but more pronounced than for simple field-changes (Table 1) was found; N_+/N_- decreased from 3·0 for magnitudes $> 5{,}000$ v/m, to a constant value of approximately 0·75 below 500 v/m. 40 per cent of the complex field-changes contained three or more sections, usually with a negative net effect and a negative first section for magnitudes < 100 v/m, and a positive start and a positive net change above 500 v/m. These very involved changes of field were particularly common during the later stages of storms.

4. Cathode-ray oscillograph results

In 1947 and 1949, oscillograph records of over 1,500 electrostatic field-changes were obtained. Some field-changes included both slow and rapid sections but many were entirely slow in character, and in general there was a sharp differentiation between fast and slow portions. Sections of intermediate slope were seldom encountered.

The terminology adopted for describing field-changes was based upon the following scheme. Any rapid section, exhibiting apparent discontinuities in slope, upon the record, at start and finish, and of a duration $\gtrsim 5$ msec was classed as an '*R*' change. Slow variations were divided in three ways; those preceding the first *R* section in a field-change were described as '*L*' changes; those occurring between two *R* elements were termed '*J*' sections; while slow variations, in field-changes not containing *R* elements, or succeeding the final *R* section in a field-change, were classed as '*S*' changes. Appleton and Chapman (1937) divided the field-change, due to a discharge removing negative electricity to earth, into three sections, the '*a*,' '*b*' and '*c*' portions, corresponding to the initial variation, the fast portion and any final change of field, respectively. The terms *a* and *b* have been replaced by Malan and Schonland (1951) by '*l*' and '*r*,' while they have also introduced the letter '*j*'; these letters are employed with broadly the same meaning in the present nomenclature.

There are perhaps objections to assigning the upper limit of duration for the *R* change at as high as 5 msec, since former observers have given much smaller values for the times occupied by any fast elements in the field-change. Records of high time-resolution show that, on Appleton and Chapman's notation, the transition from the *a* to the *b* sections is abrupt and that from the *b* to the *c* parts comparatively gradual; the *b* element may often be considered as succeeded first by a fast 'c_1' and then by a slower 'c_2' change. The apparently sharp transition on the present records taken as representing the end of the *R* element, probably defines the boundary between the c_1 and c_2 portions; thus what is now considered as the *R* section is likely to include the *b* and the initial part of the *c* change, on the previous nomenclature.

Almost all the electrostatic changes of field recorded oscillographically had total magnitudes between 0·1 and 100 v/m, and therefore were due to flashes beyond the reversal zone of radius 20 km (§ 3.2); no very close storms were studied. Characteristic examples of the field-changes recorded are depicted in Figs. 3 and 4. In these drawings, the vertical coordinate represents v/m on a linear scale, but although the field-changes are accurately delineated, no actual vertical scale is included. This is because all the forms shown have been observed with total magnitudes ranging from 0·5 v/m to 50 v/m, according to the distance of the storm, and it would therefore be misleading to specify any precise vertical scale. Figs. 7 and 8 show reproductions from the original records; sketches of each field-change are included for clarity. An upward deflection upon the records represents a positive change in field. For each time-base sweep, time increases horizontally from left to right, while successive sweeps are separated so that time also increases downwards. In the field-change sketches the ordinate is *E* the field in v/m and the abscissa *T* the time in seconds.

Figure 3. Typical electrostatic field-changes.

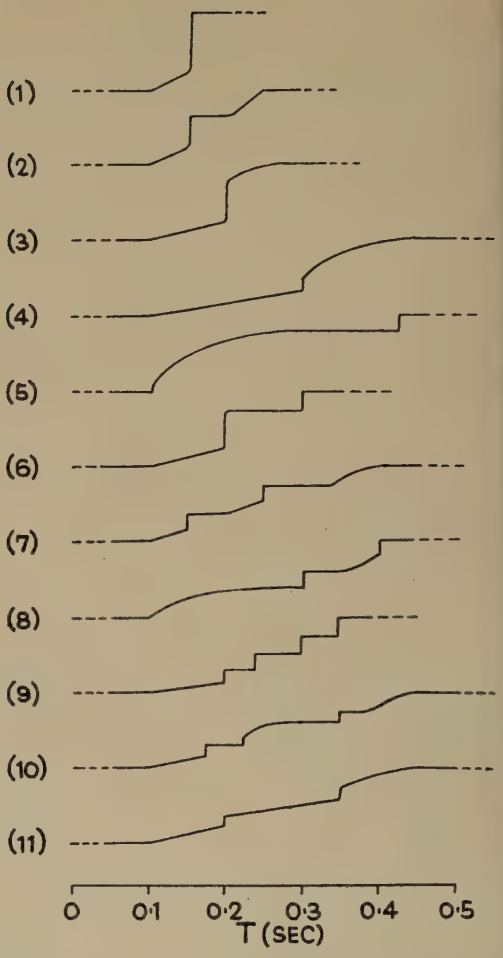

Figure 4. Typical electrostatic field-changes.

Field-changes may be conveniently separated into six fairly distinct groups, according to whether the change in field is positive, negative or complex, and contains fast R elements or not. In the present observations, slow negative changes of field and positive field-changes with rapid sections, were the most common types recorded (see § 6 for precise details).

It is clear from previous work (Chapman 1939), that if a flash reaches the ground the resultant field-change will contain an R section associated with the return-stroke, while air or cloud discharges will produce only slow changes of field. This result was confirmed by observations made during a night storm, when all the field-changes identified as due to discharges to earth contained fast R elements.

4.1. The slow negative field-change

Examples of this type are given in Figs. 3 (4-6) and Fig. 7 (1). The field-change may take place in sections interrupted by quiescent intervals (Fig. 3 (6)) or may be continuous (Fig. 3 (4, 5)); the rate of change of field is usually most rapid at the start. Fig. 5 is a histogram showing the distribution according to total duration of the 685 slow

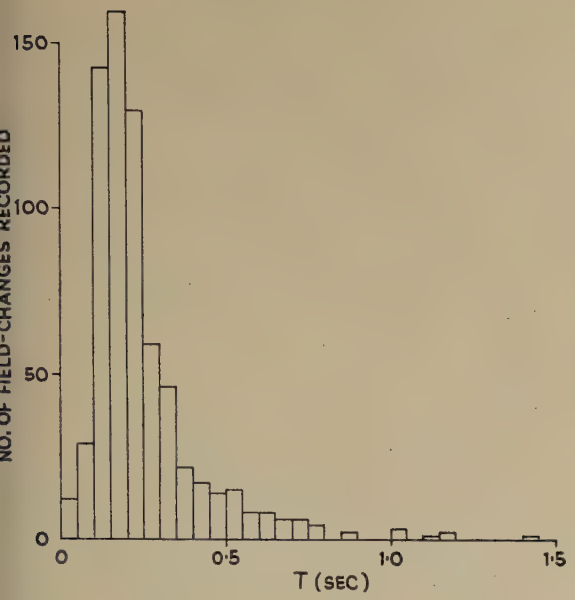

Figure 5. Duration histogram for slow negative field-changes.

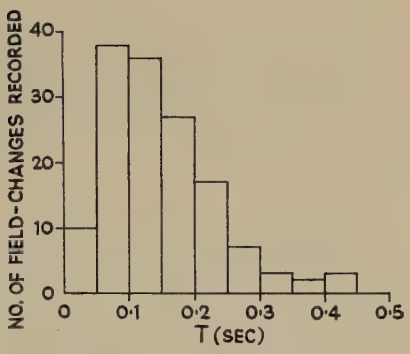

Figure 6. Duration histogram for slow positive field-changes.

negative field-changes recorded. The mean duration is 245 msec; about half of these field-changes lasted for over 200 msec and seven exceeded a second in duration. Slow negative field-changes are due to air or cloud discharges which either lower positive charge or, more probably, raise negative charge.

4.2. The slow positive field-change

Fig. 3 (1-3) and Fig. 7 (2) are illustrative examples of this type of field-change. Again, the field-change may occur in sections (Fig. 3 (3)) but the rate of change of field is generally more uniform than for the slow negative variety, and the rapid start is not so often observed. Fig. 6 is the duration histogram. The average duration, for the 143 field-changes, is 145 msec, the median 130 msec, and the maximum 440 msec; these values are appreciably less than the corresponding figures for slow negative field-changes. Slow positive field-changes are produced by flashes which do not reach the earth, and which probably involve the downward movement of negative charge.

4.3. Positive field-changes with fast elements

Field-changes of this kind are produced by flashes conveying negative electricity to ground; typical examples are given on Figs. 4 and 8. Usually, a gradual *L* rise in field is succeeded by one or more rapid *R* elements separated either by quiet intervals or by slow *J* changes, and there is often a final *S* section.

89 per cent of the 373 field-changes of this type recorded, commenced with initial *L* sections; in the remaining 41 field-changes the first detectable variation was a fast *R* element. In practice, when displacements on the record were small it was easier to detect rapid rather than slow sections. However, of the 41 field-changes, 29 contained subsequent *J* or *S* portions. It thus appears that, in comparatively rare instances, the initial *L* field-change may be extremely small, or, alternatively, be completed so quickly as to be included, under the present notation, within the first *R* element.

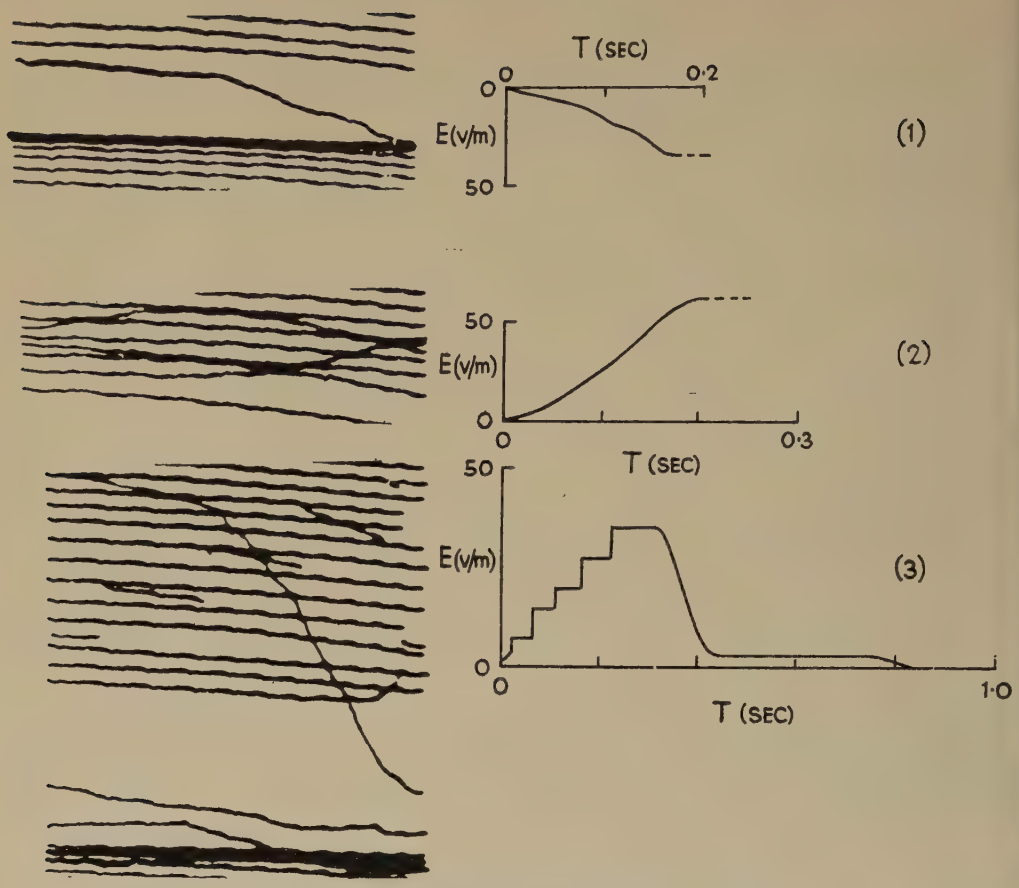

Figure 7. Typical records of electrostatic field-changes.

A flash with more than one return stroke, i.e., a field-change having more than one R element is conveniently described as 'multiple.' 74 per cent of the intervals between the R elements in multiple field-changes were found to contain no alteration in field; slow J variations were included within the remaining 26 per cent.

Of the 373 field-changes observed, 182 finished with a slow S section. These S changes were particularly frequent for flashes with only one stroke, being produced by 59 per cent of such discharges; the corresponding percentages for field-changes with two to four, and five or more, R elements, were 44 per cent and 30 per cent, respectively.

4.31. Number of strokes, time-intervals between strokes, and duration. 38 per cent of the positive field-changes with fast elements contained only one R section; the corresponding figures for two, three, four, five and six or more, strokes, were 27 per cent, 17 per cent, 9 per cent, 4 per cent and 5 per cent.

The average time-interval between successive R elements was 93 msec, and the median value 65 msec; 64 per cent of the intervals were greater than 50 msec, 26 per cent exceeded 100 msec, and 9 per cent were above 200 msec. The interval between the 'nth' rapid element, R_n, and R_{n+1}, may be termed that of order n. A systematic increase in the mean duration of the interval with increasing order was found, the average durations for intervals of order 1, 2, 3 or 4, and 5 or more, being 86, 94, 102, and 107 msec, respectively. The effect was apparent whether the intervals included J changes

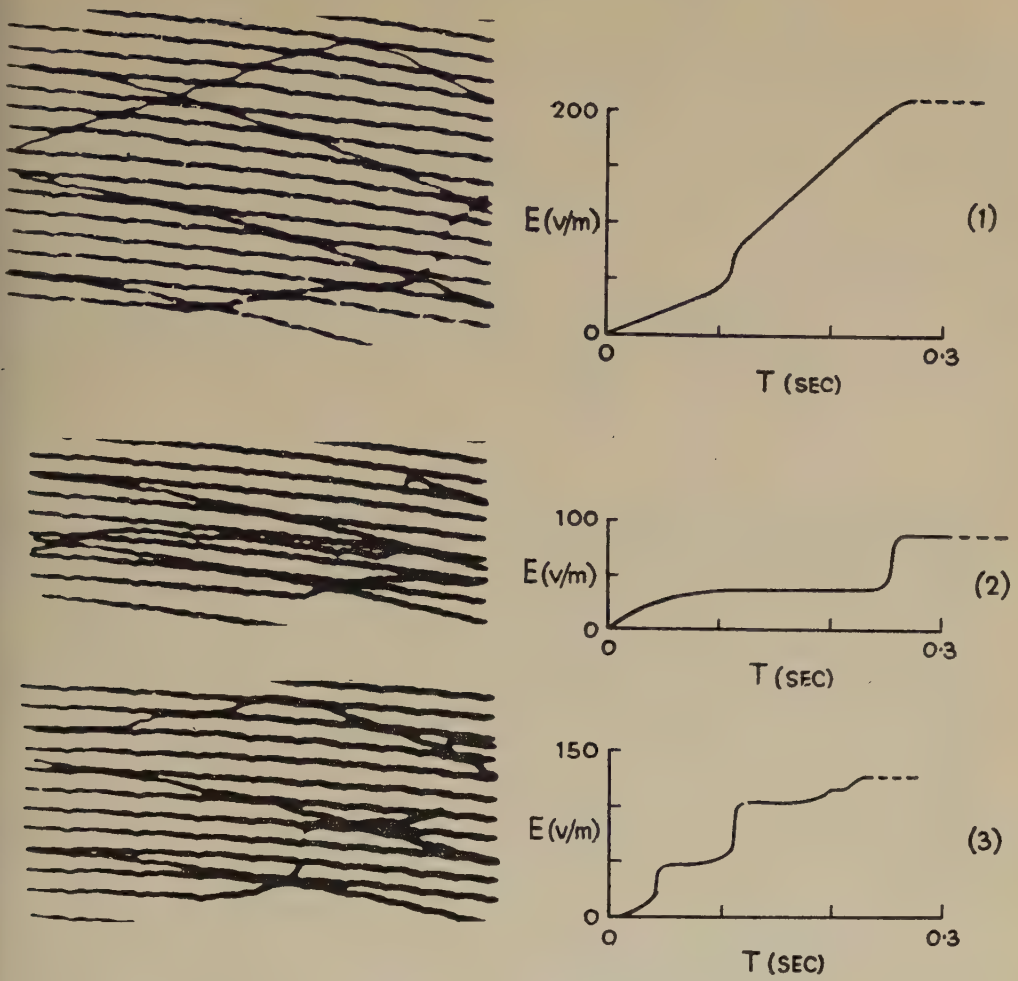

Figure 8. Typical records of electrostatic field-changes.

or not, and was due to the greater proportion of very long durations recorded for the higher-order intervals. The percentages of time-intervals exceeding 150 msec, for instance, were 9 per cent, 12 per cent, 18 per cent and 22 per cent, for orders 1, 2, 3 or 4, and 5 or more, respectively.

The mean total durations for field-changes with one, two, three or four, and five or more, *R* elements, were 150, 200, 340 and 515 msec, respectively. For all the 373 discharges the average value was 245 msec, and the median, 180 msec.

Where comparison is possible, the results in this section are in reasonable agreement, except in one respect, with former observations (Bruce and Golde 1941). The discrepancy is in the duration; the present figures exceed those previously given and, in particular, few field-changes of very short duration were recorded.

4.32. The initial L change. Two types of *L* variation preceding the first rapid *R* element could be distinguished upon the records. In the first, the increase in field was uninterrupted up to the *R* portion, while in the second variety the initial slow field-change was succeeded by a quiet part usually lasting until the rapid section, although there was sometimes a fairly short slow rise in field immediately preceding the *R* element.

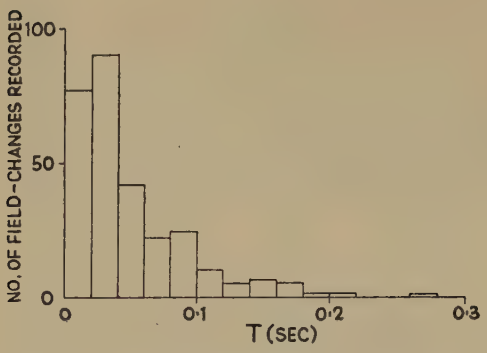

Figure 9. Duration histogram for $L(\alpha)$ leader field-changes.

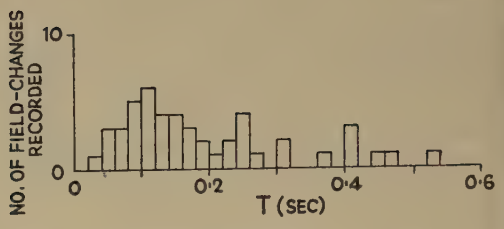

Figure 10. Duration histogram for $L(\beta)$ leader field-changes.

Somewhat similar effects have been noted by Schonland, Hodges and Collens (1938) and given the titles 'α' and 'β' by them; this nomenclature will be retained, the two kinds of initial field-change being denoted by $L(\alpha)$ and $L(\beta)$. Typical examples of $L(\alpha)$ changes are shown on Fig. 4 (1-4, 6, 7, 9-11) and Fig. 8 (1, 3), and of $L(\beta)$ variations on Fig. 4 (5, 8) and Fig. 8 (2). 284 $L(\alpha)$ changes and 48 of the $L(\beta)$ variety were recorded.

Figs. 9 and 10 are histograms for the total duration of the $L(\alpha)$ and $L(\beta)$ types, respectively. The average duration for an $L(\alpha)$ change was 50 msec, while about half these variations persisted for over 35 msec; the corresponding figures for the $L(\beta)$ variety are 175 msec (during 95 msec of which a rise in field was actually occurring), and 140 msec.

The proportion of the total change of field, due to the whole discharge, occurring during the L section, was found to be significantly higher for $L(\beta)$ than for $L(\alpha)$ variations; the appropriate percentages, for discharges containing one stroke, were 55 per cent and 40 per cent. For all field-changes, the mean value of an $L(\alpha)$ variation was 120 per cent of that in the succeeding R element, with half the $L(\alpha)$ changes producing increases in field greater than 95 per cent of that in the rapid section. $L(\beta)$ variations gave, upon the average, a change of field 240 per cent of that represented by the following R section, with a median figure of 220 per cent.

4.33. The rapid R elements. In recording, the brilliancy was usually adjusted so that the most rapid variations in electrostatic field would not register upon the photographic film. Thus, if an R element was completely recorded, it could not have included any very large rates of change in field. On the originals of the records reproduced on Fig. 8, all the R elements are just visible for Fig. 8 (1) while the R elements for Fig. 8 (2, 3) contain rapid sections which have not affected the film. This visibility criterion could be used to classify R elements; a second and less satisfactory differentiation was according to total duration.

41 per cent of the R_1 and R_2 elements contained sections not visible on the record; the respective percentages for the R_3 and R_4, and the elements of higher order were 34 per cent and 26 per cent. The total duration was less than 1 msec for 40 per cent of the R_n elements when n equalled 1 or 2, the corresponding figures being 28 per cent for n equal to 3 or 4, and 15 per cent for $n \geqslant 5$. The R_1 section appeared to be more likely to contain especially fast portions if preceded by an $L(\alpha)$ rather than an $L(\beta)$ variation; 44 per cent of the R_1 elements following $L(\alpha)$ changes included parts not recorded, the proportion for those succeeding $L(\beta)$ sections being only 26 per cent.

It is perhaps surprising that the average duration ($>$ 1 msec) of R elements, given by the observations, is so large, and it must be remembered that differences in definition

§ 4) compared with previous work exist. The frequent absence, however, of any detectble changes of field occurring very rapidly, is evidence that the major part of an R eld-change often takes place comparatively slowly. In this connection, Bruce and Golde 1941), who examined some of the records of Appleton and Chapman (1937), noted that nany fast elements when closely scrutinized consisted of a very small rapid section ollowed by a rather slower change in field.

Measurement of the relative magnitudes of the R elements, indicated that with given by 2, 3, 4, 5 or 6, and $\geqslant 7$, the changes of field due to the R_n element, were, pon the average, 125 per cent, 130 per cent, 140 per cent, 130 per cent and 100 per ent, respectively, of that represented by the R_1 section in the same discharge. The ange of variation was large; field-changes were occasionally noted in which one R section, vhich might be of any order, gave 20 per cent or 500 per cent of the mean field-change $\Sigma R_n/n$ associated with an R element for that particular flash.

4.34. The final S changes of field. Two types of slow S variation succeeding the last R element could be distinguished. In the first variety, the slow S part followed instantly fter the fast section and was uninterrupted until the steady value of the field was attained; n the second kind, the gradual field-change was interspersed with quiet periods during vhich no variation in field was detectable; such quiescent intervals often immediately ucceeded the R element. By analogy with the notation for L sections the two varieties vere termed $S(\alpha)$ and $S(\beta)$ changes, respectively. Examples of $S(\alpha)$ variations are iven on Fig. 4 (3, 4, 11) and Fig. 8 (1, 2) and of $S(\beta)$ changes on Fig. 4 (2, 7, 10) and ig. 8 (3).

The average duration of the 80 $S(\alpha)$ changes recorded was 85 msec, the median alue being also about 85 msec. The corresponding figures for the 102 $S(\beta)$ variations vere 145 msec (during 80 msec of which a change of field was actually occurring), and 25 msec; three $S(\beta)$ sections exceeded 500 msec in total duration.

A greater proportion of the S changes were of the $S(\beta)$ type when the discharges ncluded more strokes; for field-changes containing S variations but with only one R lement, 47 per cent of the S sections were of the $S(\beta)$ kind, the respective percentages or flashes with two to four, and five or more, fast steps being 61 per cent and 90 per cent.

The change in field was much the same within both the $S(\alpha)$ and $S(\beta)$ variations; in ach case, it amounted to about 250 per cent, upon the average, of that represented by he final R element.

4.35. Slow J field-changes occurring between the rapid R elements. 132 of the intervals eparating R sections contained slow J changes of field, while in 371 intervals no variation n field could be detected. For intervals including J variations in which an increase of ield immediately preceded the following R element, the average duration was 75 msec nd 61 per cent of the subsequent R sections contained very fast portions not visible on he record. The corresponding figures were 200 msec and 31 per cent for those intervals vith J changes in which a quiescent section occurred just before the R element, and 5 msec and 29 per cent for intervals without J variations.

Upon the average, the change of field within the J section was 135 per cent of that epresented by the subsequent R element. Large R sections were most likely to follow he intervals of longer duration; this was true for intervals both with and without J hanges. The proportion of subsequent R elements containing very rapid parts decreased, owever, as the duration of the interval increased, this effect being especially marked for hose intervals without J variations.

4.4. Negative field-changes with fast elements

In general, the structure of these field-changes was similar (in reverse) to that of the corresponding positive variety, and it is probable that they are to be associated with the passage of positive charge from cloud to earth; some typical examples are given on Fig. 3 (7, 8).

Only 34 field-changes of this type were recorded in all; 6 were multiple, and of the 12 intervals between R elements that they contained 7 included J variations. 82 per cent of the field-changes observed were initiated by L sections of appreciable size, these being subdivided into 53 per cent with L (β) changes of average duration 310 msec, and 29 per cent with L (α) variations of mean duration 45 msec. About half the field-changes contained final S sections; S (β) type variations were less common than the S (α) kind, but were often of very great duration the mean value being 380 msec.

In other respects, the characteristics of the negative field-changes with R elements did not differ appreciably from those for the corresponding positive type.

4.5. The moments involved in the different field-change types

Descriptions of the four varieties of simple electrostatic field-change have been given, but no assessment has yet been made of the moment changes accompanying each of the four types. Ideally, such a determination requires a single stationary storm, well beyond the reversal zone (0-20 km), generating field-changes of all four kinds, this being the only activity within recording range; this situation was approximately realised on two occasions. On the 16 July, 1949 80 field-changes were observed in a period of one hour the only thundery outbreaks being at a distance of 70-90 km, while on 14 July 1947, 213 field-changes were recorded, the activity at the time of the observations being 50-80 km from Cambridge. From the field-change data for these two storms, the average total change in moment for each of the field-change types as given in Table 2 was obtained. Analysis of all the field-changes with fast elements recorded, indicated the average percentages of the change of field taking place in the L, R, J and S sections, respectively; a sub-division has accordingly been made in Table 2.

TABLE 2.* AVERAGE MOMENT CHANGE (C KM) IN EACH SECTION OF INDICATED TYPES OF ELECTROSTATIC FIELD-CHANGE.

Type of electrostatic field-change	Change in moment (C km)							
	In L section	In first R element	In second R element	In third R element	In fourth R element	In final S sections	Intervening J sections	Total
Positive with one R element	35	29	–	–	–	25	–	89
Positive with two R elements	31	28	34	–	–	19	13	125
Positive with three R elements	27	27	31	32	–	22	22	161
Positive with four R elements	30	24	31	31	32	23	25	196
Negative with one R element	54	17	–	–	–	36	–	107
Slow positive	–	–	–	–	–	–	–	45
Slow negative	–	–	–	–	–	–	–	81

* The definition (see § 4) of an R element is important. Many field-changes with R elements do not contain S or J portions and the average moment changes given in the seventh and eighth columns are therefore correspondingly reduced. When J and S sections are apparent, the changes in moment involved are usually more than twice those indicated.

There is some discrepancy between Table 2 and the results of § 3.1, and it is probable that the procedure there adopted tends, in spite of the criterion (2b), to select larger field-changes than the typical values; this is particularly so for negative field-changes. In the two storms mentioned above, the observations showed a large spread in magnitude for both positive and negative changes of field. That for the positive field-changes was primarily due to the presence of two families, slow and including R elements, both fairly closely grouped but around widely differing average values of moment variations. The range of magnitude for slow negative field-changes was of itself extremely large. Mean and median figures for the moment involved in the field-changes from the two storms were 140 and 90 C km, and 100 and 70 C km, for positive and negative field-changes, respectively. If, however, the procedure of § 3.1 was applied to 10-min periods during these particular storms, the figures obtained for both signs of field-change approached those, 150 and 110 C km, given in § 3.1.

4.6. Complex field-changes

Of the 273 complex field-changes recorded oscillographically, 149 contained two sections and 124, three or more. Almost all the separate sections exceeded 100 msec in duration; this is twice the time resolution of the capillary electrometer, therefore few complex field-changes were not recognized as such on the electrometer records, and it is not surprising that there is good agreement between the results obtained by the two techniques. The great majority of the complex changes in field observed oscillographically, either contained positive R elements or were entirely slow in character, and often appeared to be a combination of two or more of the simple types. It is perhaps noteworthy that negative R elements were more likely to occur within a complex than in a simple change of field.

Typical examples are given on Fig. 7 (3) and Fig. 3 (9, 10); Fig. 3 (9) represents the individual form of most common occurrence. The mean duration of the complex field-changes was 420 msec and, in general, those of magnitude less than 100 v/m produced net negative effects.

4.7. Radiation fields

The transient radiation and induction fields, E_R and E_I, although unregistered upon the usual continuous records, were investigated by means of the supplementary techniques described in § 2. In records of transients of this kind, it is difficult to determine precisely what features are to be associated with E_R and E_I respectively; however, since the storms studied were beyond 30 km, while the predominant frequencies involved in the transients recorded were all above 3 kc/sec, Eq. (1) indicates that the transient disturbances were primarily due to E_R; it will therefore be assumed that the supplementary observations represent variations in E_R.

In general, the results obtained confirm and extend those of Appleton and Chapman (1937) and Chapman (1939). R elements were found to be usually accompanied by radiation fields, the average peak-to-peak magnitudes of which were 1·0, 3·0, 6·0 and 15 v/m. for R sections in which the electrostatic field-changes were 0·1, 1·0, 10 and 100 v/m respectively.

Slow field-changes, on the other hand, were often devoid of any associated radiation structure, most of the variation in field being entirely electrostatic and therefore smooth in character. No radiation fields were ever observed upon the J and S sections of the positive field-change with R elements, but radiation effects were usually found upon

the L section, and for the slow positive and slow negative types of electrostatic field-change. Even in these cases, however, the radiation structure was usually confined to the parts where the rate of change of electrostatic field was greatest; the first 10 or 20 msec of the slow negative field-change represented on Fig. 3 (5), for instance, often contained superimposed transients which were absent from the remainder of the variation in field. For slow types of electrostatic field-change, of total magnitudes 0·1, 1·0, 10 and 100 v/m, the maximum peak-to-peak values of any associated radiation fields were approximately 0·1, 0·3, 0·5 and 1·4 v/m, respectively.

An examination of the fine structure of the radiation effects accompanying slow changes indicated that they consisted of an irregular series of pulses of characteristic form. Similar pulses were noted by Appleton and Chapman (1937) on L sections, and it was now found that the pulses associated with slow negative field-changes were identical in character, allowing for the difference in sign, with those previously observed. The form of a typical pulse could be approximately represented by the expression

$$\pm (h_1^3\, t\, e^{-h_1^3 t^4} + h_2^3\, t\, e^{-h_2^3 t^4}) \quad . \quad . \quad . \quad (3)$$

where $h_1 = 3{\cdot}5 \times 10^5$, $h_2 = 1{\cdot}2 \times 10^5$, and t is in μsec, the centre of the pulse being the origin in time.

5. Meteorological factors

It has often been previously suggested that storms differing meteorologically will also differ in their electrical characteristics. No classification of thunderstorms exists that is accepted by all meteorologists, but a distinction, according to their manner of origin, is often made between two main types, ' heat or orographic ' and ' frontal.' This simple differentiation is frequently difficult or even unjustified, but in so far as possible, the storms during the recording periods were separated into heat and frontal categories.

Lutkin (1939) stated that frontal storms produce a larger proportion of negative field-changes than do heat storms. Examination of the capillary electrometer records showed that the ratio N_+/N_- of § 3.2, for field-changes less than 100 v/m, had values between 1·0 and 2·5 when the thunderstorms were of the heat category, while when the activity was frontal the range was from 0·2 to 1·0. Lutkin's observation was thus confirmed.

Another meteorological distinction first reported by Schonland (1938) was that frontal storms give more multiple flashes than do heat storms. Although sufficient material for this effect to be examined was available on only five days, it was found that upon the one day when the activity was frontal, 45 per cent of the positive field-changes with R elements contained three or more strokes, while for the other four occasions the corresponding percentages ranged from 22 to 38 per cent; the pertinent evidence,therefore, though scanty, is in agreement with Schonland.

6. The relative frequencies of occurrence of cloud and ground discharges

The field-change observations may be used to estimate the relative proportions of cloud and earth flashes that take place in a typical storm. The results of § 3.2 show that the ratio N_+/N_- of simple positive to simple negative field-changes approaches a limiting value r as the magnitude decreases. If the moments involved in negative (M_N) and positive field-changes (M_P) are equal, this limit r represents the ratio of the percentage of flashes from a given storm that produce positive field-changes (F_P) to the

ercentage giving negative changes of field (F_N). If, however, $M_P = k\,M_N$, it is easy o show that $F_P/F_N = r\,(k)^{-\frac{2}{3}}$. The precise value of k is difficult to determine. § 3.1 ndicates that $k = 1$, but § 4.5, although based on much less data, shows that the pro- edure of § 3.1 tends to overestimate M_P and M_N, and particularly the latter quantity. The best assessment of k is probably that, 1.3, obtained from the median moment figures f § 4.5. This gives $F_P/F_N = r\,(1{\cdot}3)^{-\frac{2}{3}} = 0{\cdot}8\,r$.

It has already been pointed out that r has different values for individual storms §§ 3.2 and 5) depending apparently on meteorological influences. During 1947, most f the records were for storms of frontal type, and from the value of r, 0·6 (Table 1), F_P/F_N is found to be 0·5. Thus in frontal conditions in 1947, 27 per cent of the dis- harges produced simple positive field-changes, 54 per cent gave negative field-changes, nd 19 per cent generated complex changes of field.* The 27 per cent giving positive ield-changes could be further subdivided on the basis that, in 1947, positive field-changes vith R elements were three times as common as slow positive changes of field; similar ubdivisions were made for negative and complex field-changes. These subdivisions re included in Table 3 which shows the proportion of the indicated types of field-change enerated by a typical heat storm ($r = 2{\cdot}0$), a typical frontal storm ($r = 0{\cdot}6$), and a storm epresenting the long-term average behaviour ($r = 1{\cdot}1$). It will be noted that in this verage storm, 39 per cent of the flashes include R elements, i.e., go to earth, while 1 per cent are in the cloud or air.

No great accuracy is claimed for the figures of Table 3. Some of the processes nvolved in their derivation were merely approximate, while several complicating factors, or instance, the considerable difference in the average moment involved for the two inds of positive field-change, the legitimacy of applying arguments based on the 1947 lata to the long term observations, etc., were neglected.

TABLE 3. PERCENTAGES OF ELECTROSTATIC FIELD-CHANGES OF DIFFERENT TYPES GENERATED BY THE INDICATED KINDS OF STORM.

Type of field-change	Frontal storm	Heat storm	Storm representing the average long-term behaviour
Positive with R elements	20	35	27
Slow positive	7	14	11
Negative with R elements	2	1	1·5
Slow negative	52	29	40·5
Complex with positive R elements	6	11	8
Complex with negative R elements	1	1	1
Complex with R elements of both signs	2	1	1·5
Slow complex	10	7	9·5

Wormell (1953a) has given a table indicating the proportions of flashes of various ypes in thunderstorms observed at Cambridge; this was also based upon the oscillographic bservations of 1947 and 1949. No attempt was made however, as has been done in his Section, to incorporate the information upon long-term behaviour that can be derived rom the capillary electrometer results. In consequence, slight discrepancies exist between Table 3 and that of Wormell.

* For distances > 20 km, the proportion of complex field-changes appears to be about 20 per cent irrespective of the meteoro- gical type of storm, but the distribution between slow field-changes and those with R elements, alters.

7. Conclusion

A long series of observations of the electrostatic field-changes due to lightning discharges has been described. These observations have yielded detailed information upon the moments involved in the changes of field and the precise manner in which the field-changes take place. No endeavour has, however, been made in the foregoing text to compare in detail the results obtained with previous findings, or to discuss their implications and reconciliation with the probable processes taking place in thunderstorms; this is attempted in the following paper.

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The development of lightning discharges

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Summary

The interpretation of experimental observations, described in a preceding paper, of the electrostatic field-changes due to lightning flashes, is considered. It is found that almost all the recorded field-changes can be most simply explained in terms of leaders carrying negative charge and originating in the lower part of a thundercloud. Those that reach the ground are succeeded by return strokes. It is considered that both leaders and return strokes consist of a central core surrounded by an envelope of corona; this corona concept, originally advanced by Bruce, leads to a coherent picture of the phenomena observed.

1. Introduction

An extensive experimental investigation of the changes of electric field at the earth's surface associated with comparatively close (distance < 150 km) thunderstorms, has been described in a preceding paper (Pierce 1955, referred to as I). This work has yielded a comprehensive picture of the electrostatic field-changes produced by lightning discharges. The present paper is concerned with the implications of the results given in I and their relation to the probable processes occurring in thunderstorms.

The terminology adopted in I will be retained in this paper.

2. The initiation of lightning discharges

The general outline of charge distribution in a thundercloud in temperate latitudes is well known. It consists of a positive charge, 20 to 40 coulombs in size, centred at a height of about 6 km, and a negative charge of approximately the same magnitude centred at a height of about 3 km, that is, near the freezing level. This distribution was originally indicated by field-change measurements (Wilson 1916, 1920), definitely established from the alti-electrograph work at Kew (Simpson and Scrase 1937, Simpson and Robinson 1941), and subsequently confirmed by the aeroplane observations of Gish and Wait (1950). Small regions of positive charge towards the base of a thundercloud have also often been noted, and Schonland and Malan (1951) have suggested that these are due to point discharge from the surface of the earth.

All lightning flashes begin as leader streamers which are initiated when local conditions produce a field large enough to cause breakdown. High local fields may be occasioned by the distortion of precipitation elements, the mutual approach of minor charged centres, etc. Breakdown is most likely to occur where the general field is greatest, that is, according to Ross Gunn (1948), within the region where the temperature does not differ greatly from 0°C. In horizontal flights at the approximate altitude of the freezing level Gunn has observed high fields which change rapidly with distance; these fluctuations are probably due to local pockets of positive charge which may, as suggested by Malan and Schonland (1951b), play an important part in the initiation of leaders. Conditions are thus most favourable for the generation of leaders near the freezing level, that is within the negatively

charged region, and since the vast majority of leaders that reach the earth carry negative charge (I, § 6), it is reasonable to assume that the most probable zone for their initiation is in the negatively charged domain at a height of about 3 km. From here, leaders carrying negative charge will radiate. Those reaching the ground will give the familiar leader, return stroke, sequence, while upward leaders will not be followed by a return stroke but will merely distribute negative charge among the upper positive charge. This general concept of a central nucleus from which leaders originate is supported by visual observations of thunderstorms; flashes to earth from the lower part of a cloud are familiar to all, while the reports of experienced observers (Simpson 1909; Schonland, Malan and Collens 1935) suggest that the majority of discharges within the cloud pass from base to summit rather than in the opposite direction.

3. Comparison of the present experimental results with previous observations

The most important feature of the results described in I was the discovery of the great magnitude and long duration of the slow sections in the electrostatic field changes. There seems no doubt that the records obtained were genuine representations of the field changes occurring, and that the previous underestimation, e.g., by Appleton and Chapman (1937) of the slow effects was due to deficiencies in recording technique, in particular the lack of adequate low-frequency response. The observations of Malan and Schonland (1951a) on the *J* and *S* sections of the positive field-change with fast *R* elements are in agreement with those of I in demonstrating the importance of the slow portions in the field-changes.

Photographic techniques as used by Schonland and his collaborators also yield durations for many sections of the discharge which are considerably shorter than those indicated by the field-change observations of I. For instance, Schonland, Hodges and Collens (1938) found that α leaders in a discharge to ground last for about 10 msec, while the duration of the L (α) change (I, § 4.32) is 50 msec. It is possible that not all the L (α) stage is to be associated with the development of the main downward leader. This however does not seem likely, because when radiation pulses (I, § 4.7) are superimposed upon slow negative, slow positive, or L field-changes, three types of field-change that might plausibly be associated with the advance of a negative leader in virgin unionized air, the pulses are almost always most evident near the start of the change in field. It is well known that these radiation pulses are coincident with the bursts of luminosity known as steps that occur during the progress of a leader; the photographic evidence shows that steps tend to be more pronounced during the early stages of downward leaders. Thus, phenomena of simultaneous origin occur towards the commencement both of the L field-change and of the downward leader, and it appears probable that all of a slow change in field is to be associated with the movement of a leader; Malan and Schonland (1951a) in explaining the *J* section assume that this is so.

This discrepancy between the duration of leaders to earth, when assessed photographically, and of the L field-changes is difficult to explain; it may be due to the fact that much of the leader is obscured by the cloud; the photographic technique may also tend to select the brighter, i.e., faster leaders while ignoring, for instance, a β type leader (Schonland *et al.* 1938) in which all the initial bright steps are within the thundercloud. Considerable movements of charge may also occur without any associated photographic effects, and in this connection Davis and Standring (1947) when studying discharges to barrage balloons found that currents of up to 500 a can flow without any accompanying luminosity or thunder.

4. Leader processes

4.1. Velocity

The mean duration of the L (α) section in a positive field-change due to a discharge to earth is 50 msec (I, § 4.32). Assuming the associated leader originates at a height of 3 km and follows a direct path to ground, the corresponding average velocity is 6×10^6 cm/sec. Furthermore, slow negative field-changes last, on the average, for 250 msec (I, § 4.1). If, again, the leader streamer starts from the main negative charge at a height of 3 km, the vertical path upwards cannot greatly exceed 6 km before the top of the thundercloud is reached; this implies a mean vertical speed of $2{\cdot}4 \times 10^6$ cm/sec. Normally, the leader channel will not be entirely vertical but will contain inclined sections; the estimates given for the vertical component of leader velocity should therefore be increased, perhaps by as much as 100 per cent, in order to obtain the actual mean speed of the leader tip.

Even so, the values are appreciably less than those found for negative leaders by Schonland and his collaborators. From photographic evidence, Schonland, Malan and Collens (1935) give the minimum effective velocity of a leader tip as $1{\cdot}0 \times 10^7$ cm/sec, with a most probable value of $1{\cdot}5 \times 10^7$ cm/sec, and a mean figure of $3{\cdot}8 \times 10^7$ cm/sec; in a more recent publication Schonland (1953), on the basis of field-change observations, quotes 4×10^7 cm/sec as the most frequent value of leader velocity. The lower figures derived in the present work are not entirely inconsistent with other relevant data. Laboratory investigations on long sparks give a range of from 10^5 to 10^8 cm/sec according to the external circuitry and general experimental conditions, Malan and Schonland (1951a) find a speed of 3×10^6 cm/sec for positively charged leaders associated with J sections, and it seems reasonable to expect that leader velocities will, if anything, be greater for positive than for negative leaders, since in the former case electron movement will be in a converging and not a diverging field. Schonland (1938) has estimated that the minimum velocity of leader advance lies between $1{\cdot}8 \times 10^7$ cm/sec and $1{\cdot}4 \times 10^6$ cm/sec; a value approaching this lower limit is suggested by some of the slow negative field-changes of very long duration.

4.2. Currents and conditions within the channel

The average currents in the leader channel are of the order of 100 a or more. Table 2 (I), for example, indicates that a moment change of about 30 coulomb km occurs during the L section of a flash taking negative electricity to earth. Accepting the height of the negative charge feeding the leader as 3 km, it follows that the variation in moment may be imagined as produced by the lowering of 5 C of negative charge to earth; since this takes place in 50 msec the average current involved is 100 a. Peek (1929) has indicated that with currents of this size a low-voltage high-conductivity arc regime must exist within the leader, the tip of which will therefore be at the same potential except for the small drop along the arc, as the charge-centre feeding the leader streamer. As the leader advances, there will be a considerable potential difference between the more forward sections of the channel and the surrounding air, and lateral corona currents will develop.

This general picture is supported by the laboratory work of Allibone (1948), Saxe and Meek (1948), and Komelkov (1947a, 1947b, 1950); they find that a typical leader consists of a central bright narrow core, surrounded by a sheath of comparatively large radius but less intense ionization, and that the tip of the main leader is preceded by a cloud of fine filamentary corona streamers.

Bruce (1944) was the first to recognize the importance of lateral corona currents in the lightning discharge. He developed his ideas analytically, comparing his conclusions with the data of Appleton and Chapman (1937); these do not represent average conditions, and it is therefore of interest to re-examine the corona-current concepts on the basis of the results of I.

4.3. Lateral corona currents

It is well established that the radial corona current from a high voltage wire is proportional to the square of the potential difference between the wire and the surrounding space, provided the voltage considerably exceeds that required to initiate corona. Accepting that this law applies to the gaseous leader and following Bruce (1944), the corona current I_c from a length dl of the leader channel may be written,

$$I_c = k\left\{V_0 + \int_0^l X\,dl\right\}^2 dl \qquad . \quad . \quad . \quad (1)$$

where k is the corona constant per cm of channel, V_0 is the potential discontinuity at the start of the leader between the channel and the adjacent space, l is the distance along the channel (assumed vertical) of the element of length dl from the initiation point of the leader, and X is the general field in which the leader is advancing. In (1) the drop in potential along the arc existing within the leader channel is neglected. The current I flowing in the channel at the starting point of the leader must be sufficient to maintain the lateral corona currents and is therefore given by

$$I = \int_0^L I_c\,dl \qquad . \quad . \quad . \quad . \quad . \quad (2)$$

where L is the length of the leader. With v representing the velocity, L may be expressed as

$$L = \int_0^t v\,dt \qquad . \quad . \quad . \quad . \quad . \quad (3)$$

Combining (1), (2) and (3), the expression for the change of moment M in time t is

$$M = \int_0^t \int_0^L 2kl\left\{V_0 + \int_0^l X\,dl\right\}^2 dl\,dt \qquad . \quad . \quad . \quad (4)$$

As a first approximation it is convenient to take X and v as constant and $V_0 = 0$; (4) then reduces to

$$M = \frac{kX^2 v^4 t^5}{10} \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

For the L (α) section in the flash transporting negative charge to earth, the total variation in moment is about 30 C km; the change of field is accomplished in 50 msec, and assuming the streamer starts from a height of 3 km, v is 6×10^6 cm/sec. The cloud-charge distribution of Simpson and Robinson (1941) entailed a potential difference between the centre of the main negative charge and the ground approaching 10^8 volt in magnitude, while the records of Ross Gunn (1948) also show that values of this order are attained between the earth and the point where the general field changes from negative to positive. Bruce and Golde (1941) found that a charge of about 1 C is neutralized in the first 0·1 msec of the upward travel of the return stroke; the results of I confirm this figure. Assuming that 1 C of charge is deposited during the leader upon a channel with radius, 8 cm, equal to that (Schonland 1953), of the return stroke, and using a formula for the channel capacitance given by Goodlet (1937), the result for the total potential

difference involved is again about 10^8 v. Thus a reasonable estimate for X is 300 v/cm extending over a distance of 3 km. Using (5) and the above figures, k is found to be $0{\cdot}8 \times 10^{-18}$. (5) assumes $V_0 = 0$, and its precise value is difficult to assess but Bruce (1944) has pointed out that for the Empire State Building the potential difference required to initiate an upward leader is 2×10^6 v. Repetition of the preceding analysis with $V_0 = 10^7$ v, an estimate probably considerably in excess of the true value, only changes the calculated figure for k from $0{\cdot}8 \times 10^{-18}$ to $0{\cdot}6 \times 10^{-18}$. If analysis is applied to the slow negative field-change produced by an upward leader the value obtained for k is again approximately 10^{-18}.

No great precision is claimed for this figure. A change, for instance, in the height of origin of the leader from 3 km to 2 km, would increase the value obtained for k by a factor of 5. The chief uncertainties are in the data for the total length of the leader, which affects v, and for X; average figures for M and t, on the other hand, at the instant when the leader reaches the earth are well established from the results of I. Bruce (1944) using the observations of Appleton and Chapman (1937), obtained 4×10^{-17} for k, the present information suggests that this figure is too high by a factor of at least 5.

The peak current flowing in the leader channel may be estimated from (2); for a leader associated with an $L\,(\alpha)$ type field-change it is about 600 a. McEachron (1939) recorded currents of up to 500 a during leaders from the Empire State Building; the agreement is excellent, but not necessarily of great significance since discharges from the Empire State Building are undoubtedly abnormal in character.

Almost all the records described in I were obtained with the storms at some distance from the observing station, and in consequence the variations in the field E directly represent the changes in electric moment M. Expression (5) indicates that with X and v constant, M is proportional to t^5. Some of the $L\,(\alpha)$ sections show this kind of variation but in the majority of cases the change in M with time is less abrupt and may be written as $M = Ct^n$ where C is a constant and n lies between one and three (e.g., I, Fig. 8 (3)) while the $L\,(\beta)$ changes actually show a decrease in dM/dt with increasing t (e.g., I, Fig. 8 (2)). Several influences neglected in the preceding simple analysis can produce these effects. Firstly, X is not constant with respect to l; for the case of the downward leader it almost certainly decreases as l increases. Secondly, since charge is redistributed in feeding the leader, at any particular height the general field X will vary with time. Again, the velocity of the leader is not constant; there is some evidence (Flowers 1943) to show that the speed of advance is directly proportional to the general field at the tip which in turn will depend upon l and t. Furthermore, it has been taken that, in the arc conditions existing within the leader, any drop in voltage along the channel is small; Komelkov (1947a, 1947b), however, finds a gradient of 60 v/cm, and an additional complication is that this gradient will vary according to the current passing and thus, at certain parts of the leader, approach the value of X. Any precise analysis giving due weight to these four factors is difficult, but it is easily possible with simple assumptions as to decreasing X with increasing l, for instance, to obtain forms for the L changes which approach closely to the variations actually recorded.

Bruce (1944) was criticized upon the grounds that he tended to regard the leader channel as similar to a sharply-bounded wire surrounded by corona. Although it is true that the inner boundary of the corona zone is not rigidly defined, the very presence of the core to the channel seems to suggest that it only varies within narrow limits. Perhaps a more serious defect in the type of corona analysis given above is the fact that the fundamental Eq. (1) implies that for a constant X and V_0, the corona current over a length dl of the core at a distance l is independent of time. This will not be so because electrons involved in the corona will quickly form negative ions of comparatively low

mobility, particularly outside the corona zone; these cannot move far during the total duration of the leader and will form an effective space-charge blanket tending to inhibit the corona process.

Goodlet (1937) first pointed out that the outer boundary of the corona zone is defined by $2q_l/r_l \simeq 100$ ESU, the minimum ionizing field, where q_l and r_l are the charge per unit length and the maximum radius of the corona zone at a distance l, respectively. Assuming that the whole leader channel, core and corona sheath, is a good conductor it is possible by adapting an analysis originally given by Davis and Standring (1947) for a balloon cable connected to the earth, to show that q_l and r_l both increase with l. This result probably applies to actual channels also, although it is then likely that arc conditions and good conductivity only exist within the core, and that there are radial gradients in the surrounding zone approaching those 6-10 kv/cm observed by Komelkov (1947a) for corona streamers.

4.4. Steps

The observations described in I (§ 4.7), show that the radiation pulses associated with brightly luminous steps are often completely absent from, or occur at widely separated intervals upon, both the L variations and the slow positive and slow negative types of electrostatic field-change. Whenever extensive stepping takes place it accompanies sections in which the rate of change of electrostatic field is high, but most leaders appear to be of a continuous character for all or by far the greater part of their duration. Thus for British thunderstorms, the stepped effect is apparently not the fundamental phenomenon suggested by the records of Schonland and his collaborators in South Africa. Accordingly, no distinction, such as that of Schonland (1953) between the continuous pilot ' streamer ' and the occasional step ' leaders ' that reinforce it, has been made; the term ' leader ' as employed in this paper refers effectively to the continuous pilot process and not to any step phenomenon.

Several explanations of step formation have been advanced. The recombination theory of Meek (1939) envisages steady currents of less than 1 a in the leader channel, but in order to explain the observed electrostatic field-changes currents of some hundred amperes are required. Uncertainties exist in the glow-arc transition theory put forward by Bruce (1944) as to the exact current at which the transition occurs; laboratory experiments are misleading, since electrode effects are then important.

In a recent paper Schonland (1953) has pointed out that the current passing during a step is approximately 1,000 a. The results of I (§ 4.7) indicate that in a step radiation-pulse a moment change of about 10^{-3} C km occurs in 1 μsec; assuming the corresponding current flows throughout the length of the channel, the current is again of the order of 1,000 a. Schonland considers that arc conditions exist within the step leader, and that it and the pilot streamer have the same large radius of several metres; this is that of the outer boundary of the corona zone. In consequence, he obtains the current density in the leader, both for step and steady currents, as being between 10^{-3} and 10^{-2} a/cm^2; this is much less than the 10 a/cm^2 found by von Engel (1929) for arcs carrying currents of approximately 100 a. The electron densities involved in the Schonland picture of the leader are only 10^9 to 10^{11} cm^{-3}. Since suggestions have been made (Flowers 1943) that luminosity is dependent on electron density, and that photography is difficult for densities $< 10^{11}$ cm^{-3} (Loeb 1939), these low values appear barely sufficient to account for the luminosity of the steps.

It is interesting to note that if the advancing negative leader encounters a minor concentration of positive charge, a phenomenon resembling a step will almost certainly occur. The leader is preceded by a cloud of filamentary corona streamers (Allibone

1948); one of these, which will ultimately form the core of the leader, bridges the gap to the positive charge, and as soon as this takes place a high current is established in the narrow filament which will, in consequence, suffer a rapid expansion (Higham and Meek 1950). At the same time the current will be dropping to the steady value required in the leader core to maintain the corona losses. Since a decrease in total current and an increase in channel diameter both tend to diminish the intensity of the light emitted (Flowers 1943), the channel will become less luminous and the characteristic brilliancy variation visible as a bright step confined to the region near the leader tip will be produced. Pockets of positive charge will exist at the base of the thundercloud (Schonland and Malan 1951) and in the air as the result of point-discharge from the earth's surface, and the approach of a negatively charged leader towards such pockets, will produce the effect described above. Such a phenomenon would, however, be of comparatively random incidence and difficult to reconcile with the extreme regularity of the formation of steps as recorded in South Africa.

5. Discharges to earth

5.1. Flashes carrying negative charge to ground

These discharges are responsible for the positive field-changes with fast R elements; when the negatively charged leader reaches the earth a rapid return stroke follows with an associated R field-change. Pronounced differences in the rate of change of field in the R elements occur (I, § 4.33), and suggest that considerable variations in channel conductivity, or in the availability of charge from the earth, exist at the instance the downward leader reaches the ground. In particular, slow R sections tend to follow L (β) variations, and it appears that in the final stages of the leaders associated with these variations, at least, appreciable recombination must take place within the channel, in spite of the high average currents found in general for leaders.

The moment-change occurring shows that a negative charge of some 8 C is distributed in and around the channel during the leader process; the nearer the ground, the wider the corona zone and the more charge is probably deposited. Integration of current-time records indicates that about 1 C of charge presumably present on or near the leader core is quickly neutralized during the first 0·1 msec of the return-stroke ascent. Although it seems definite that the corona zone will have a radius of several metres (Schonland 1953), varying along the length of the leader, it does not appear that the leader core and return stroke channel are necessarily of widely differing dimensions. Komelkov (1947a) has observed a leader core diameter of 0·2 cm, but since the current density in an arc is 10 a/cm^2 (von Engel 1929) and taking arc conditions and a current of 100 a to be present in the lightning leader core, the corresponding diameter is approximately 4 cm; it is possible that the character of the comparatively freshly established core of Komelkov may be intermediate between arc and spark. A spark channel attempts by expanding to attain a limiting current density of 10^3 a/cm^2 (Flowers 1943) and, by considering this expansion and their own results, Higham and Meek (1950) estimate the diameter of a return stroke carrying a peak current of 20,000 a to be between 3 and 4 cm after a time comparable with that of the return stroke phase; if a continuing current of 100 a then follows, the eventual conditions attained will be of an arc character with the corresponding diameter (4 cm).

After the return stroke has passed up the channel, arc conditions tend to be established in its wake. Thus, especially at the greater heights, the channel will be at a positive potential with respect to the surrounding air; this will still contain the negative charge evolved during corona discharge from the downward leader and predominantly concentrated towards the lower sections, together with the remnants of the negative charge

originally initiating the leader. Corona processes will therefore commence. Near the ground they will perhaps continue until the positive charge drawn from the earth and distributed from the channel is sufficient to neutralize the existing negative charge, eliminate the potential difference, and thus inhibit the corona. Towards the upper parts of the channel, the sheath of negative charge will be small and the positive potential large as regards the adjacent air; the corona will thus develop a positive space charge surrounding the channel. The accompanying field-change will be of the final S (α) type (I, § 4.34); from the nature of the mechanisms envisaged, the approximately exponential form (e.g., I, Fig. 8 (2)) of many of these variations is easy to understand. There is of course no certainty that the positive corona discharge will continue until true equilibrium is established and the corona entirely ceases, indeed loss of conductivity or interruption of the channel will probably prevent this being attained; the frequent absence of any pronounced final S field-change (I, § 4.3) suggests that these latter effects are not uncommon.

After a return stroke, positively charged leaders advance from the initial path of the flash towards other charged regions in the cloud (Bruce and Golde 1941); if one of these is reached, a rapid ' dart ' to earth and a second return stroke follow. These intervening or junction leaders are responsible for the slow J changes of field (I, § 4.35) and have been studied in detail by Malan and Schonland (1951a) in South Africa. By considering the reversal distance effects observed, they were able to show that the J changes were definitely due to the ascent of positive charge rather than to negative charge moving downwards. They found that for flashes from medium distances (12-30 km), 64 per cent of the intervals between R elements contained positive J variations, 7 per cent included negative J changes, while in 29 per cent there was no alteration in field. From their results, Malan and Schonland (1951a and 1951b) deduced that in a normal thundercloud the negatively charged regions involved in a multiple discharge are disposed in a vertical column, and are tapped in succession by positively charged junction leaders moving upwards through about 0·7 km. In the field-change work of I, 578 intervals between positive R elements were noted; of these 158 contained positive J changes, 10 included negative J sections, while the vast majority, 410, showed no appreciable J variation. It thus seems that in Great Britain most of the intervening positively charged leaders moved horizontally, and that the cells in a British thunderstorm tend to develop several negative charges at roughly the same height. An interesting point is that the succeeding R elements contain faster sections when immediately preceded by J variations (I, § 4.35); this effect is similar to the distinction between R elements following L (α) and L (β) variations, and seems again to imply appreciable recombination and loss of conductivity within some leaders.

When a J variation is present the average associated change of moment is 40-50 C km; since this is greater than that in an L change, while the vertical distance advanced is probably considerably less for a junction than for the initial leader, further support is obtained for the view that two fundamentally different types of leader carrying charges of opposite signs are involved. Positive currents exceed negative for corona from charged wires (Bruce 1944); if the same effect applies to leader channels, the greater moment found for J as compared with L variations is understandable. The vertical extent of a junction leader is 0·7 km, and hence the charge involved is roughly 30 C. The existence of the reversal distance effect for J field changes (Malan and Schonland 1951a) implies that the positive charge feeding the upward leader does not flow from the earth during the progress of the leader but must have been established at a height comparable with the length of the original discharge channel before the junction leader was initiated. It must have moved from the ground during the return stroke and succeeding continuing-current phase

of the flash, and in order for 30 C to pass from the earth into the channel, integration of current-time records shows that conducting contact must have been maintained at the ground for several msec, at least. Thus it appears essential for the first stroke of a multiple discharge to be of the 'hot' or 'continuing current' rather than the 'cold' variety of lightning.

Considerations of capacitance indicate that it is unlikely for so large a charge as 30 C to be carried on the channel itself. Most is probably distributed around the upper channel by corona processes, and does not disperse or recombine immediately but forms a nucleus for the instigation of positive leaders. The motion of the leaders is to some extent revealed by the form of the final S field-change. An S (α) type variation will be mostly due to corona processes from the original channel, possibly associated with continuing luminosity (Malan and Collens 1937), and will imply no development, or horizontal movement, of any subsequent leaders, while the later phases of S (β) changes may well be produced by leaders travelling upwards but not attaining a new negative charge centre.

5.2. Flashes bringing positive charge to earth

These discharges produce negative field-changes with R elements and are of relatively rare occurrence (I, § 6). They differ from flashes to earth giving positive field-changes in three important respects. The L section is normally of long duration and L (β) type; there is seldom more than one R element; while most of the moment change takes place in the leader phase. All these effects suggest that the discharge is from the upper positive charge in the thundercloud to earth, and this deduction is supported by the more frequent occurrence of negative field-changes with R elements in the later stages of a storm, when, as the result of lateral displacement of the main charges, such flashes might be expected to be most common. The large change of moment during the leader process is probably chiefly due to the great height involved but increased corona from positive channels may be a contributory factor. A feature of the observations of I was that complex field-changes, in which the first R element was negative and the second positive, were recorded more often than multiple negative field-changes; thus after the initial return stroke any intervening junction leaders appear more likely to tap a negative than a positive charge centre.

Besides the two main charge centres in a thundercloud, Simpson and Robinson (1941) consider that a positive charge capable of discharging to earth exists near the base of the cloud. This charge may be responsible for the field-changes with L (α) type sections. Negative field-changes with R elements are, however, uncommon, and only a small proportion of them have L (α) variations. Any lower positive charge, although perhaps playing a major role in the initiation of leaders, is therefore of minor importance as far as field-changes are concerned.

On the records of I for the positive field-changes with R elements the J section, which is certainly associated with the advance of a positively charged leader, has never been definitely observed to include superimposed radiation structure. Radiation pulses are, however, occasionally present upon the L changes, and this indicates that stepping may be predominantly a phenomenon of negative leaders. In this connection, Schonland and Malan (1954) have suggested that the stepped leaders from the Empire State Building, were of negative and not of positive polarity as previously reported. The evidence as regards the L sections of negative field-changes with R elements, i.e., stages due to positive leaders, is scanty and inconclusive. Pulses sometimes appear to occur on the L change just before the R element, but since they only precede the return stroke by a short time, it is uncertain whether they are to be associated with the positive leader or with the subsidiary negative leaders that may develop upwards from the ground under the influence of the main downward stroke (Golde 1947).

6. Discharges not reaching the earth

Flashes in the cloud or air produce field-changes that are completely slow in character, and, hence, are entirely due to leader processes. In many of the slow field-changes, the rate of change of field is greatest at the start and then decreases steadily. The form is often very similar to that of an $L(\beta)$ field-change; it may be explained by some of the effects referred to in § 4.3, and probably implies that the leader is advancing with lessening speed in a rapidly diminishing general field.

Slow positive field-changes are probably produced by downward leaders from the main negative charge that do not reach the ground before being exhausted by corona processes; the moment-change in these field-changes is accordingly comparable with that occurring during the leader phase of the normal discharge to earth (I, § 4.5). It seems unlikely that slow positive field-changes can be due to upward leaders from the positive charge centre near the summit of the thundercloud. Gish and Wait (1950) consider that the discharge from the top of a thunderstorm is in the nature of a small steady current; this would not be observed as causing a field-change with the recording technique described in I. Furthermore, in view of the general increase of conductivity with height, there appears to be little to prevent the vertical extent of any upwardly directed lightning flashes being considerable, and the resulting field-change will then be complex at most of the observing distances concerned.

Slow negative field-changes have already been tentatively associated with leaders ascending from the main negative charge through the body of the thundercloud. These may be expected to produce changes of moment of 100 C km or more according to the vertical distance travelled. Although visual observations indicate that flashes within clouds are predominantly directed upwards, and the negative charge near the base is the most favourable zone for the initiation of leaders, it is possible that positive leaders originating in the upper positive charge and moving downwards may cause slow negative changes of field. Such leaders would be the ultimate development of those associated with the $L(\beta)$ sections observed on negative field-changes with R elements, the leaders being arrested in this case before the ground is reached. The great range of change in moment found for slow negative field-changes may result both from considerable variations in the vertical height involved and from their being generated in two different ways.

7. Complex field-changes

Many of the complex field-changes observed may be explained by considering them as produced by the combined effects of two leaders occurring almost simultaneously; these would each separately produce a simple field-change. For instance, the common slow type with three sections depicted in I (Fig. 3 (9)), may be attributed to a leader progressing upwards from the main negative charge followed by a downward leader from the same charged region. At first, the field-change is completely negative; when the downward leader commences the rate of change of field that it occasions is greater than that due to the ascending leader, but since the duration of the descending is less than that of the upward leader (I, §§ 4.1 and 4.2), the effect of the former soon vanishes and the final part of the complex field-change is negative. As slow negative field-changes exceed slow positive field-changes on the average (I, Table 2), the net result is negative. When fast R elements occur in a complex field-change, they imply, of course, that one leader reaches the ground.

It is not always necessary to assume the existence of two separate leaders. If a leader is at one stage moving upwards and at another downwards, a complex field-change will be produced. Complex field-changes, in which a slow negative J variation separates

two positive R elements, are almost certainly due to the positively charged junction leader following a downward, and not, as is more usual, an upward or horizontal path. Again, a type of complex field-change often noted near to a discharge, consists of a slow negative variation succeeded by a typical positive field-change with an R element. This variety is probably caused by a single negatively charged leader descending and reaching the earth; at first, by virtue of the reversal distance effect, the field variation will be negative. Upward leaders to the ionosphere, if they occur, will also as indicated in the preceding section, give complex field-changes.

8. Conclusions

A discussion has been given of the results described in I and of the manner in which they conform with the probable mode of development of the lightning discharge. It has been shown that almost all the field-changes recorded can be explained in terms of negative leaders radiating from the main negative charge in the lower part of the thundercloud. Those that reach the earth are followed by return strokes. Conditions within the leaders and return strokes have been examined, and it is found that both probably consist of a central core surrounded by an envelope of charge due to corona. This corona concept, the importance of which was originally recognized by Bruce (1944), leads to a coherent and comprehensive picture of the phenomena observed with no major inconsistencies.

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The coalescence between small water drops

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Summary

An experiment is described for measurement of the rate of coalescence in a cloud of droplets of nearly equal size. It is found that the value of the collection efficiency is unexpectedly high, and a physical explanation of the effect is given. The influence of electric charges on the droplets has been observed, and it is shown that increases by a factor of up to 20 in the collection efficiency are possible.

1. Introduction

It is now reasonably well established that coalescence of cloud droplets is an important mechanism leading to the formation of rain, but doubt still exists about the droplet size at which this process becomes effective. Langmuir and Blodgett (1946) have considered the problem of a large drop falling through a cloud of uniform droplets, and have given values of the collection efficiency E of the order of 0·95 for a drop of 1 or 2 mm diameter passing through droplets 40 μ in diameter. The collection efficiency decreases with the decreasing size of the drops, and tends towards zero for droplets less than 10 μ diameter. These values have been verified experimentally by Gunn and Hitschfeld (1951) for the case of a 3·2 mm drop falling through a cloud of smaller droplets, that is, under conditions corresponding to those in a cloud which is already raining.

In order to obtain an understanding of how rain forms by coalescence in the first place, it is also important to investigate the situation which exists in clouds which have recently formed by condensation. These are known to consist of a large number of small droplets with a comparatively small range of sizes. An experiment has therefore been designed to measure the collection efficiency of droplets of nearly equal diameters. The basis of the method is that drops are introduced into the centre of a vertical wind tunnel in which the air velocity is just sufficient to lift them upwards. Should coalescence occur, a drop of double the mass is formed, which then falls downwards and emerges from the bottom of the tunnel. If the size distribution of the injected drops, their rate of delivery and the upward air velocity are known, it is possible to calculate the collection efficiency from the rate of arrival of the double drops at the base of the tunnel. The method has three important advantages over one in which the drops are allowed to settle in still air. The concentration of the drops is many times greater than that obtained by free settling. Secondly, the drops remain in the vertical part of the tunnel for several seconds, and this is equivalent to a substantial increase in the free-fall tunnel length. Thirdly, drops which have undergone coalescence can very easily be separated from the vastly greater number which have not, because they are falling through the tunnel. If necessary, they can be collected at the bottom.

As a result of initial work with this apparatus, two observations of importance to theories of rain formation have emerged and these will be described in this paper.

2. The wind tunnel

The design of the wind tunnel adopted is shown diagrammatically in Fig. 1. Because of the desirability of using saturated air to avoid evaporation difficulties and of the need to run the working section at atmospheric pressure, a completely enclosed air circuit

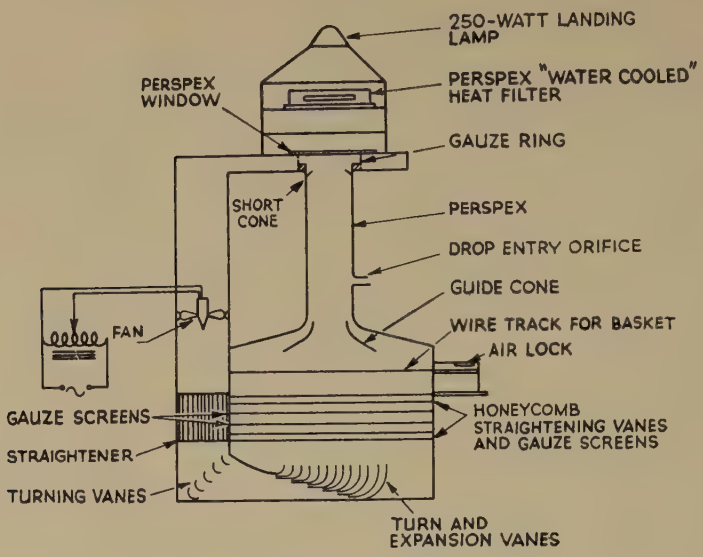

Figure 1. Vertical section of wind tunnel.

was decided upon. Unfortunately, this type of construction leads to difficulties in maintaining a streamlined flow, because the air makes a second passage through the tunnel before turbulent eddies have died down. Extreme precautions had therefore to be taken to achieve streamlined flow and to maintain it for an adequate working period. In this, a great deal of useful advice was obtained from the Aeronautical Department of the University of Sydney.

A brief description of the principal features of the tunnel is as follows. The circuit consists of a working section, a return duct and a settling box. The working section is a perspex tube 1·3 m high and 0·3 m in diameter, the return duct is about 0·3 m square and 2·7 m high, and the settling box is a 1·2-m cube. The air is driven by a fan in the return duct, downwards through a straightener and around a 90° turn into a constant-area duct which ends in a 4 to 1 expansion incorporating another 90° bend. This feeds the settling box which again represents an area increase of 4 to 1. The settling box has horizontal partitions of wire gauze and aluminium straighteners to break up large scale turbulence. The air leaves here through a throat into the working section which finishes abruptly, the flow turning against a flat wall and thence through a 5-cm long gauze cylinder. This section is faired into the return duct where the air again passes through the fan.

On one side of the working section is a small orifice through which drops of the requisite size are injected into the tunnel. As no air can be allowed to flow through this opening, the remainder of the tunnel was carefully sealed off. Under these conditions, the pressure in the settling box attains a level slightly higher than that in the laboratory.

The truncated cone of celluloid just below the top of the working section was found necessary in order to catch drops which tended to fall back down the boundary layer existing near the walls of the tunnel. The inside of the perspex was coated with a wetting agent ' Lissapol ' to prevent charges accumulating on the perspex, and to permit visual observations a 250-w aircraft landing lamp was arranged to shine vertically down the working section. The heat in the beam was largely removed by a water filter.

In order to perform the experiment it is necessary to have a plentiful supply of drops of nearly uniform size. A convenient method of generating these is by means of the spinning disk described by Walton and Prewett (1949). In this device, a stream of

water is fed on to the centre of a smooth disk spinning at a high speed, and a supply of very uniform droplets is flung tangentially from the disk. If the device is properly designed and adjusted, the drops are of remarkably uniform size and in general no drops are present which differ from the mean diameter by more than 5 or 6 per cent.

When in operation the air flow in the wind tunnel is remarkably steady. The practical difficulties in realizing the experiment all hinged on the problem of obtaining a uniform droplet distribution above the sampling volume for the duration of the experiment. With the rather elaborate precautions to damp out turbulence which have been taken the tunnel will run for about 15 min virtually without small-scale turbulence; this gives sufficient time for a single observation. The velocity profile of the tunnel is extremely flat, as can be seen when turbulence is allowed to build up, and a drop wanders about horizontally. The boundary layer becomes apparent only within about $\frac{1}{2}$ in. of the walls.

However, even when small-scale turbulence is absent there is an occasional swirling motion which runs up the tunnel and this accounts for the drops sometimes being swept out of the field of view of the camera. Notwithstanding this, the drops form a distinct column about 5 in. in diameter in the centre of the tunnel, and careful visual examination shows no tendency for the column to spread out as it goes up. As the drops are supplied at a constant rate uniform concentration is ensured.

3. The collection efficiency of drops of nearly equal size

Some early measurements of collection efficiency were made by collecting, below the working section, those drops which had coalesced. It was immediately obvious that the collection efficiency was considerably greater than unity, but difficulty was experienced in obtaining reliable quantitative values because of variations in the concentration of droplets across the width of the tunnel. It was therefore decided to use an alternative method in which the drops were photographed in motion in the tunnel. This gave a measurement of the number of original droplets moving upward and the number which had coalesced, moving downward. This method was free from the objections to the collection method, and, in addition, gave useful information about the mechanism of coalescence.

It was also found in the early experiments that remanent electrostatic charges on the drops could influence the collection efficiency. Precautions were therefore taken to prevent induced charges appearing on the drops, and a clearing field was applied to extract any charged drops before they entered the tunnel. It is tolerably certain that this process was effective and that charges on the droplets were not influencing the present results.

The photography of the drops was accomplished by illuminating a strip of the tunnel with an alternating current arc. This strip was photographed at right angles to the light beam by means of a camera in which the photographic film moved at uniform velocity in the horizontal direction. A very high light intensity was required and a 3·5 kw carbon projection-arc was found to be satisfactory. When operated on 50 c/sec the arc lamp produced an intense flash each 1/100 of a second. The photographs were taken with a modified oscilloscope camera on 35-mm film. In this way a photograph was obtained of a known volume of the cloud, and an upward-moving drop appeared on the moving film as a dotted line inclined upwards at roughly 45° to the horizontal. Similarly, a drop which had coalesced and was falling downwards gave a dotted line which sloped downwards from the horizontal. A single exposure of 1/25 sec therefore showed on the same photograph all those drops which had not coalesced prior to the exposure as moving upwards in the field of view, while the smaller number which had

coalesced were moving downwards. In addition, from the slope of the lines and the light-pulse repetition frequency, the individual velocities of the drops could be determined. The drop concentration was obtained directly by a determination of the volume photographed and by a count of the number of drops therein.

Table 1 gives the drop counts used to estimate the value of E given below. Some frames and parts of others were rejected because turbulent swirls in the tunnel had swept the column of drops out of the light beam. The remaining drops, however, should be a random sample of conditions in the drop column and hence this selection should not prejudice the answer.

TABLE 1. DROP COUNTS USED TO ESTIMATE E

Frame number	Number of big drops	Number of small drops	Length of frame counted (in.)
1			0
2	1	120	1·35 (total frame)
3	0	132	1·35
4	0	98	0·66
5			0
6	3	205	1·35
7	0	55	0·50
8			0
9	0	117	1·0
10	1	102	1·35
11	2	91	0·85
12			0
13	1	127	1·35
14	0	81	1·35
15	1	114	1·35
16	0	124	1·35
17	0	140	1·35
18	1	105	1·35
19	0	82	1·35
20	0	81	1·35
21	2	157	1·35
22	1	113	0·95
23	0	119	1·35
24	0	99	1·35
25			0
26	0	95	1·35
27			0
28			0
29			0
30	3	145	1·35
31	0	70	1·35
32	0	80	1·35
33	1	51	0·75
34	1	102	1·35
35	0	107	1·35
36	2	147	1·35
37	0	96	1·35
38	1	96	1·35
Totals	21	3251	37·11

The collection efficiency may be expressed as follows : the chances of a randomly selected drop having an approaching and not a receding drop next beneath it is $\frac{1}{2}$; thus the probability of a randomly selected drop having a collision in time δt following selection is

$$\delta P = \tfrac{1}{2} \frac{E\pi D^2 nv}{4c} \delta t$$

where E is the collection efficiency, D the drop diameter, n the number of small drops counted, v the velocity of approach of the drops, and c the volume of the space containing the drops.

This probability applies to each drop in a column of height H. Thus over unit rea in the column the collision rate is

$$\frac{\delta P}{\delta t} \times \frac{n}{c} \times H = \frac{\pi E D^2 n^2 v H}{8c^2}.$$

The observed value of the collision rate is NV/c, where N is the number of double rops and V is the settling velocity of the double drops.

Therefore

$$E = \frac{8NVc}{\pi D^2 n^2 v H}.$$

nserting the following values determined experimentally, $N = 21$ drops, $V = 10{\cdot}8$ cm sec^{-1}, $= 4{,}610$ cm^3, $D = 1{\cdot}54 \times 10^{-2}$ cm, $n = 3{,}251$ drops, $v = 1{\cdot}11$ cm sec^{-1}, $H = 76$ cm.

$$\text{gives } E = 12{\cdot}6.$$

To obtain an estimate of the error of this result, the accuracy of each measurement s required. The method of measurement and the approximate errors involved are iven below.

The largest error is in sampling the 21 double drops and if a Poisson distribution s assumed this is about 22 per cent.

The average settling velocity was found from measuring the velocity of double drops n the photographs. This has an estimated error of 10 per cent.

The volume photographed occupied 37·11 in. of film 1 in. wide. The magnification vas measured from a photograph of a rule in which 2 in. of rule occupied 0·783 in. of ilm. This, together with the measured thickness of the light beam of 1·16 in., gives he required volume. The error in using average dimensions of the space involved is egligible and other errors are estimated at 3 per cent.

The diameter of the drops was measured, with an error of about 2 per cent, from amples collected on an oiled slide.

The number of small drops per frame gives a distribution which leads to a standard rror of 5·5 per cent.

The velocity of approach is obtained from Fig. 2. This shows a velocity histogram neasured from a 1/10-sec exposure. The difference in slope between each track and

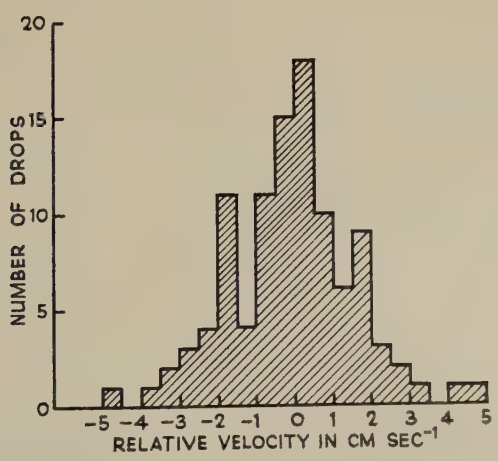

Figure 2. The distribution of fall velocities in the droplets injected into the tunnel.

the nearest track beneath it has been measured and is plotted as a velocity. The height of the ordinate is thus proportional to the probability that the next drop below a drop selected at random, will have that velocity. The average velocity of an approaching drop, ignoring the receding drops, as obtained by smoothing this curve, is 1·11 cm sec^{-1} with negligible error.

The height of the water drop column above the sampling region is 76 cm with perhaps, 1 per cent error.

If each measurement is considered to be an average value plus a random normal variate of zero mean and standard deviation as given above, and if products of the random variates are small enough to be ignored, then the values obtained for E will be a mean value plus a variable formed by summing the contributions of each error. This variate will have zero mean and variance equal to the sum of the variances of each of its components.

On this basis, the error in E is about 27 per cent, that is

$$E = 12{\cdot}6 \pm 3{\cdot}4.$$

It appears therefore that, contrary to expectation, the chances of drops of nearly like diameter coalescing are extraordinarily high.

An insight into why this should be so comes immediately from an examination of the flash photographs of drops in motion in the tunnel. Occasionally an actual collision occurs during a quarter-second exposure and the tracks of two upward-moving drops are seen to merge and then accelerate downwards. A trajectory derived from one such photograph is shown in Fig. 3 in which the successive positions of two drops 'a' and 'b' are shown during the exposure time. They coalesce at a point O and the subsequent motion of the double drop is shown by c. An important feature of the photograph is the fact that the lower drop 'a' maintains a perfectly straight path until the instant of collision, but the upper drop is accelerated downwards prior to collision. The corresponding curve of velocities derived from Fig. 3 is given in Fig. 4. Acceleration of the upper drop begins when it is still separated from the lower drop by about forty diameters. Twelve such collisions have been observed and all show essentially the same phenomenon. In order to obtain an explanation of the drop motion it is necessary to refer to a diagram of the flow pattern about the drop. As a drop of diameter 160 μ has a Reynolds number of 6 and since there is not, to our knowledge, any published solution to the flow equations at this Reynolds number we will use the flow pattern in Fig. 5 which refers to a Reynolds number of 20.

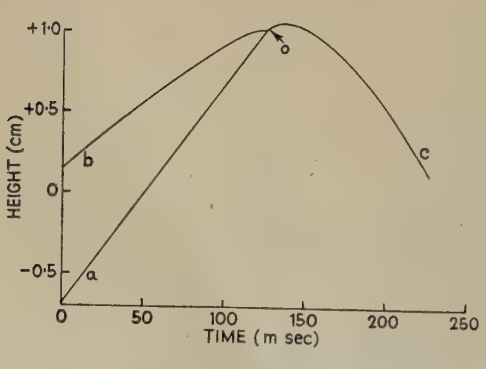

Figure 3. The motion of two drops 'a' and 'b' about 160 μ diameter immediately before and after a collision.

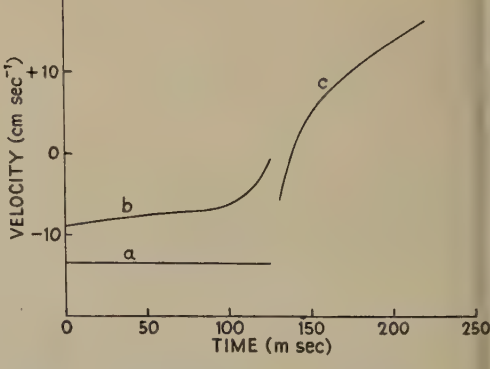

Figure 4. The velocity-time curve for the trajectories shown in Fig. 3.

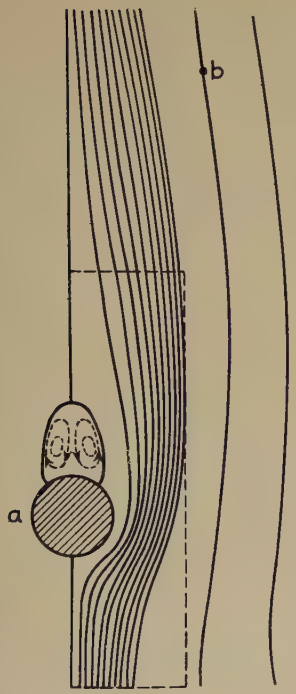

Figure 5. The streamlines about a cylinder for Re = 20. The figure inside the dotted rectangle is taken from Thom (1933) and has been extended by us to illustrate the discussion.

It appears then that the mechanism of collision is as follows. The airflow is rapidly divergent in front of the drop and only slowly approaches the axis again after the drop has passed. This results in a flow velocity behind the drop of much smaller magnitude than the main stream velocity and with a horizontal component towards the axis. When a drop at 'b' approaches 'a' its terminal velocity tends to undergo a change equal to the velocity difference of the air at its present position and that of the air a long way off. This difference is down towards 'a' but with a pronounced slope towards the vertical axis through 'a'. It is this acceleration which figures so prominently in the photographs. The sideways motion gives rise to the large collection efficiency since the disturbance in front of 'b' is too restricted to influence 'a' to move away. Thus 'a' is collected by 'b', the two drops coalesce and the combined drop accelerates away to its new terminal velocity.

It is of interest to note that the essential feature giving rise to this high collection efficiency is the asymmetry of the flow patterns around the drop. Stokes's solution is symmetrical, and gives the correct drag, but this is largely fortuitous. The serious inadequacies of his solution at appreciable distances from the sphere have often been pointed out, as by Burgess (1916) and Goldstein (1929). Indeed, it does not seem at all unlikely that appreciable asymmetry of flow occurs around *cloud* droplets even though their drag force is given very closely by Stokes's law.

The enormous arithmetical work necessary to examine this problem theoretically has been undertaken by Mr. T. Pearcey and his staff of this Laboratory. The present indications are that the simple theory based on non-accelerating flow patterns for isolated spheres gives trajectories and cross-sections remarkably close to those observed with actual drops

4. Effect of electrostatic charges on collection efficiency

Reference was made earlier to the effect of electrostatic charges on the collection efficiency of drops. It has already been shown theoretically by Cochet (1951) that electrostatic charges on small drops can influence the collection efficiency. Owing to its importance

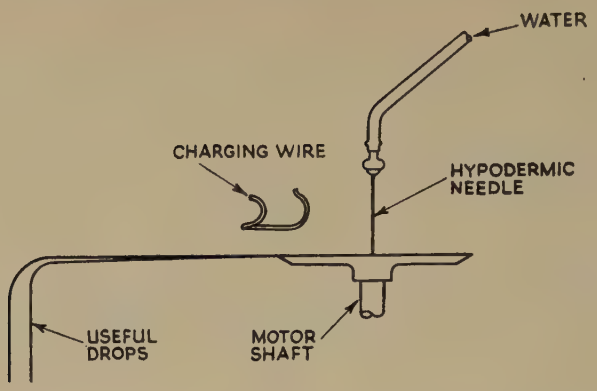

Figure 6. The droplet generator and mechanism for inducing electrostatic charge.

in theories of rain physics, this effect was made the subject of further measurements. In order to carry out these measurements, a charging wire of tinned copper was fixed along the arc of a circle just beyond the edge of the spinning disk as shown in Fig. 6. If a fixed potential was applied between the disk and the wire, the drops acquired a charge of opposite polarity as they broke away from the disk. If an alternating potential was applied to the wire, drops with equal and opposite charges were introduced into the tunnel.

To obtain a measure of the rate at which coalescence was taking place in these experiments, the drops which emerged from the bottom were collected on a microscope slide smeared with a viscous oil. The drops were then photographed and their diameters measured. Before being applied to our problem, this technique, developed by Golitzine (1950), was carefully checked by catching drops on an optically flat surface and squeezing them to a known thickness (about 5 μ) with another glass flat. The areas of the resultant irregular blobs of water were measured by photomicrography and the spacing between the flats determined by means of interference fringes and air-gauging measurements of the aluminium-foil spacers. The diameter of the original spherical drops was determined from these measurements and they checked within 1 or 2 per cent with those taken in oil samples during the same exposure. It was found that if measurement of the droplet caught in oil was delayed for more than half a minute, evaporation took place, influencing the accuracy of the result. Therefore, in all the measurements reported in this paper, the drops were photographed within half a minute of the sample being taken.

To enable drops falling from the tunnel to be sampled, the glass sampling slide covered with oil was carried on a wire frame which slid along two wires stretched across the settling box (see Fig. 1). When the frame reached the centre, a lid opened to expose the slide to the falling drops. After a given time, the lid was closed and the frame wound back by a small winch into an air lock on the side of the settling box. The inner door of the air lock was then closed, the outer one opened and the slide removed for immediate photography.

The individual measurements of the precipitation rate of coalesced drops made in this way are accurate to about 10 per cent. Calculations of the value of the collection efficiency from these figures were, however, open to considerable doubt because of the great variations in droplet concentration, due to turbulence and converging air streams, when the drops are falling through the throat of the tunnel. In the present series of measurements, therefore, only relative values are given, namely the ratio of charged to uncharged precipitation rates.

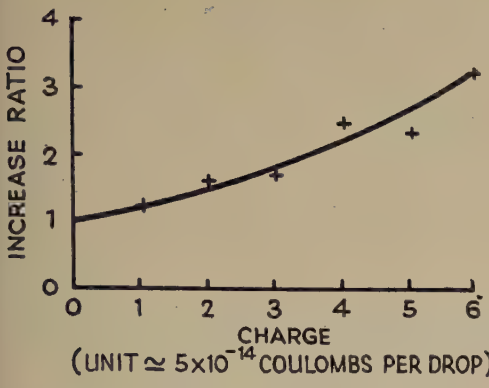

Figure 7. Increase in collection efficiency with unlike electrostatic charges in the drops. Drop diameter 130 μ, precipitation rate 2·32 drops per frame per second.

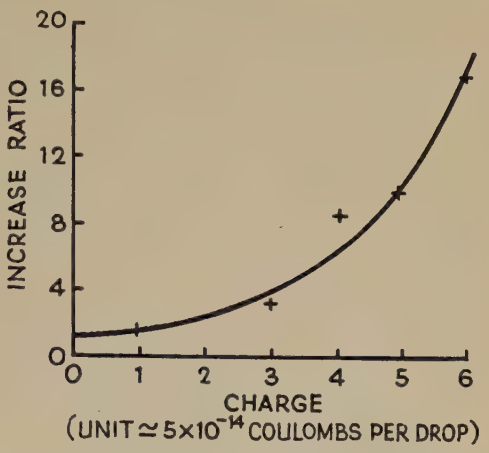

Figure 8. Increase in the collection efficiency with unlike charges. Drop diameter 130 μ, precipitation rate 0·33 drops per frame per second.

A curve showing the effect of increased electrostatic charges on the drops is given in Fig. 7 and shows that opposite charges of the order of 10^{-13}coulombs per drop can double or treble the collection efficiency of 130 μ drops as compared with uncharged drops.

A second curve taken for the same sized drops at a lower drop concentration is given in Fig. 8 which indicates that increases in collection efficiency in the ratio of 20 to 1 can occur under these conditions. This is probably owing to the fact that at higher concentrations a larger proportion of uncharged drops coalesce in any case and the effect of an electrostatic charge is proportionally less.

The effect of charging all drops to the same polarity appears to be a complete inhibition of the coalescence process. Charging some drops and leaving some uncharged does not produce an appreciable change in the average collision rate.

5. Discussion of results

The results obtained in the present investigation might have considerable impact on present concepts of rain formation by the coalescence process.

(1). The coalescence of droplets falling freely in the atmosphere has so far been considered mainly in terms of what happens in front of a falling drop. Here the flow is divergent and there is a tendency for a large drop, in falling, to push aside smaller droplets in its path. This leads to values of collection efficiency less than unity.

The present investigation brings out the importance of the flow field behind a drop, and suggests that, at least for drops of nearly equal size, the most important part of the coalescence process takes place in the wake of a falling drop. Here the flow is convergent and a drop in this region tends to move in radially behind the leading drop and at the same time accelerate down its wake and collide with it.

(2). In most treatments of coalescence to date (see, for example, Langmuir (1948), Houghton (1950) and Bowen (1950)), it has been assumed that a cloud of nearly equal droplets is in equilibrium and that coalescence will not commence unless some drops of a larger size are present or are introduced into it. The expression they use for rate of

growth includes the relative velocity term $(v - u)$. This implies that drops with the largest fall velocity are those likely to grow at the fastest rate, and that the rate of growth is small or zero for droplets with small relative velocities.

The present results show that, at least in the size range 150 μ and upwards, there is a high probability of drops of nearly equal size coalescing.

(3). The present results give a simple physical explanation of an observation relating to the size distribution of raindrops which has been the subject of controversy for many years. Defant (1905) reported some occasions on which the sizes of raindrops tended to be grouped about masses m, $2\,m$, $4\,m$, $8\,m$, etc. This has been successively confirmed and denied by a great number of subsequent workers. An explanation which was put forward at the time was based on the hydrodynamic attraction of drops of equal size falling alongside one another. It has been shown, however, that under these conditions the attractive force is negligible and that this could not be the explanation. Meanwhile, no alternative suggestion has been made and the whole question has remained undecided.

It appears from the present results that the explanation is simple. When two drops of nearly equal mass are falling near one another, the trailing drop is accelerated radially downwards behind the one which is leading. When it has fallen through the wake it then coalesces with it. This mechanism can operate for radial distances of one and a half diameters and for as much as forty diameters behind any given drop leading to very high values of collection efficiency.

(4). Another result of the present investigation is that electrostatic charges on cloud droplets have an important bearing on the coalescence rate. Charges of the magnitude found in thunderstorms can increase the collection efficiency of drops by a factor of as much as twenty times. This may profoundly modify conclusions on the possibility of droplets growing to rupture sizes in thunderstorms and so setting up chain reaction and charging processes. It is also of importance as a possible procedure for the artificial stimulation of rain from suitable clouds.

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Lee waves ahead of a warm front

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Summary

A lee-wave flight provides some observational evidence in support of a theoretical approach to the wave phenomenon and leads to a discussion on the nature of lee waves ahead of a warm front.

1. Introduction

Although lee waves in the British Isles are by no means uncommon, systematic exploration of them is difficult; field work cannot normally be planned to coincide with particular waves. In fact, the best opportunities of investigation into actual wave phenomena are usually unexpected and therefore unorganized; they are useful but cannot always be fully exploited.

Such an opportunity arose at Camphill, the Peak District site of the Derby and Lancashire Gliding Club, on 16 July 1954 when, at 10 GMT, Mr. G. O. Smith and the author flew a two-seater glider into a pronounced but unexpected lee wave ahead of an approaching warm front.

The intention had been to fly across the fresh westerly wind blowing up and over the steep escarpment which, as shown in Fig. 1, forms the western boundary of the flying field, but, by the time the sailplane had climbed to a height of 250 m above Camphill, flight conditions had become so smooth that a wave in the airflow aloft was suspected. No positive wave form in the ragged low cloud overhead could be discerned, but past experience had revealed some vague association between waves and the lack of turbulence now experienced. The suspicion proved correct, for on heading almost into wind the sailplane soon began to rise at a steady rate of 3·3 m/sec. With an airspeed of 60 km/hr in a slightly cross-wind direction it was easy to maintain a position close to the western edge of a large patch of stratocumulus overhead. Flight was now so remarkably smooth that the aircraft could almost be left to fly itself as it soared steadily upwards until, 10 min later, it had gained a height of 2,000 m.

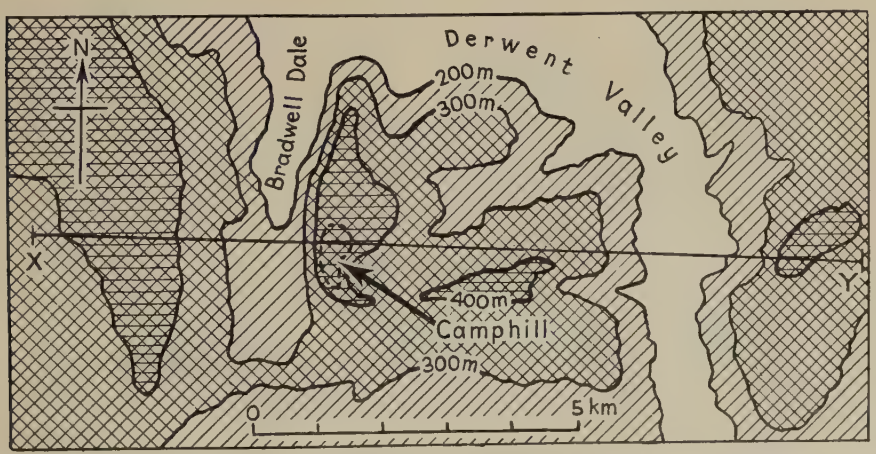

Figure 1. Sketch map showing the topography in the vicinity of Camphill. A broken line encloses the flying field, while the straight line XY indicates the cross-section referred to in the paper.

From this height there was no difficulty in discerning the unmistakable wave form in the cloud below. A cross-section of this cloud together with the high-ground profile is included in Fig. 2. To the west stretched 7/8 stratocumulus, the top of which appeared practically flat as far as the eye could see. The wave cloud over Camphill lost its smooth hump-backed contours beyond the northern and southern extremities of the 4 km-long escarpment while, at about 8 km to the east, another well-defined but smaller wave cloud stood out from an irregular pattern of 4/8 stratocumulus.

Unfortunately there was insufficient time available to prolong this particular flight, but the persistence of the wave was demonstrated by a Swedish sailplane which, one hour later, flew into the altostratus at 2,400 m.

After the flight it was decided to determine whether or not the observations made would support a theoretical approach to the phenomenon; there happened to be an ample supply of meteorological data available. Regular daylight observations had been recorded at the meteorological office temporarily set up at Camphill and an upper-air sounding to the 300-mb level had been made locally at 08 GMT by a Spitfire aircraft.

The theoretical technique used to obtain the airflow described in the following sections is that due to Scorer (1953) and is elaborated in his work on mountain airflow.

The nature of the observations made and the mathematics involved do not permit a strictly quantitative test to be made but the commentary may at least show a method of tackling a wave problem and shed some light on the type of waves to be expected ahead of a well-defined warm front.

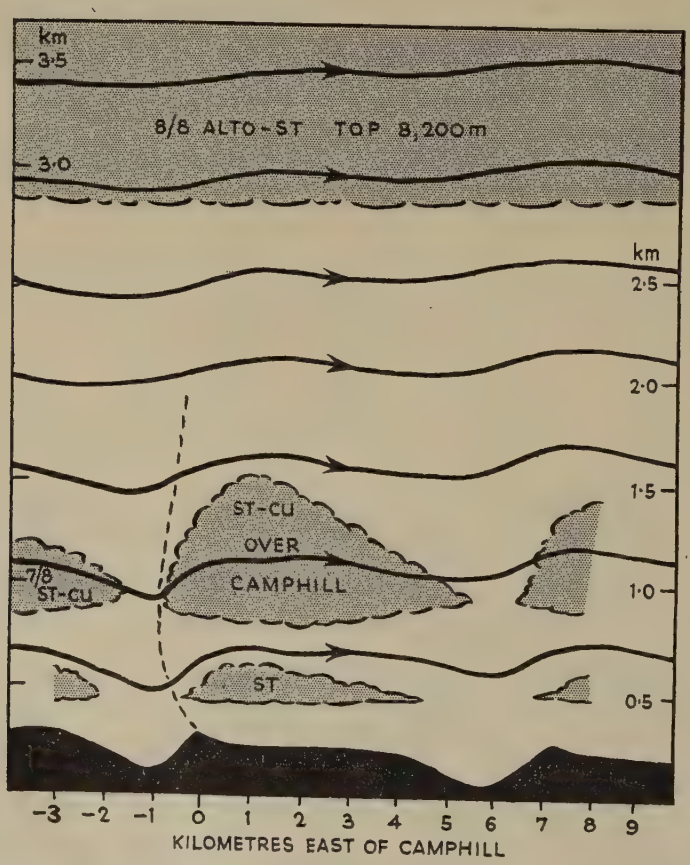

Figure 2. Computed streamlines for 10 GMT, 16 July 1954 are shown together with the observed cloud and local topography at Camphill. The described flight path is indicated by a broken line rising from Camphill.

2. The flow past a ridge or valley

The most significant factor to be considered in lee-wave problems is the variation with height of a parameter, l, given by

$$l^2 = \frac{g}{u^2 T}\left(\frac{\partial T}{\partial z} + \Gamma\right) - \frac{1}{u}\frac{\partial^2 u}{\partial z^2} \quad . \quad . \quad . \quad . \quad (1)$$

where g is the acceleration due to gravity, u is the wind component across whatever ridge or valley is to be considered, z is the height, T is the absolute temperature and Γ is the dry or wet adiabatic lapse rate (whichever is appropriate). As in many lee-wave calculations, the second term on the right is small enough to be neglected in this case.

Upper winds and temperatures in the vicinity of Camphill at 10 GMT on 16 July 1954 can be deduced from the 02 and 14 GMT Liverpool radiosonde reports and the 08 GMT Camphill upper-air sounding. These deduced winds and temperatures, which are shown alongside cloud observations in Fig. 3, allow us to consider the troposphere as three distinct layers, namely :

(i) An adiabatic layer from the ground up to the 855-mb level. In this layer $l = 0$.

(ii) A stable layer from 855 mb to 690 mb in which l is practically constant at $1{\cdot}15\ \text{km}^{-1}$.

(iii) A high layer in which l is approximately $0{\cdot}39\ \text{km}^{-1}$, both in the saturated air from 690 mb to 340 mb and in the drier air above. It is important to note the significance of the cloud layer; calculations show that if this air were unsaturated the resulting increase in stability would considerably lessen the likelihood of strong waves at lower levels.

Scorer's method enables us to determine the airflow past a two-dimensional ' idealized ' ridge (or valley) whose height, H, is given by

$$H = \frac{a^2 b}{b^2 + x^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where x is the horizontal distance from the crest of the ridge and a and b are arbitrary lengths determining the dimensions of the ridge.

He achieved this by considering solutions of the equation

$$w'' + (l^2 - k^2)\, w = 0 \quad . \quad . \quad . \quad . \quad (3)$$

where w is the vertical wind component, k is the wave number and primes denote partial differentiation with respect to height (z).

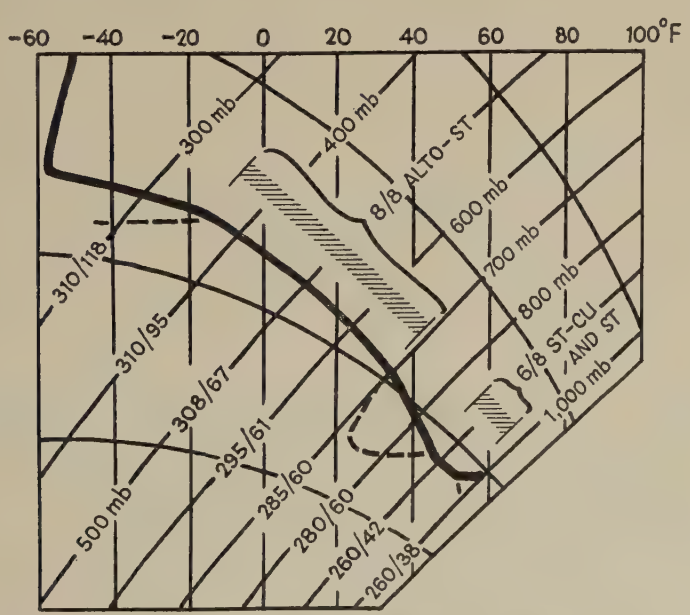

Figure 3. Upper winds and temperatures deduced for Camphill for 10 GMT, 16 July 1954. Full and broken lines indicate the temperature and dew-point curves respectively. Cloud data are based on several aircraft observations. Winds are given in degrees and km/hr.

It is convenient to consider the top of the middle stable layer as the zero of the height scale and write the appropriate solutions of Eq. (3) in the form

$$\left.\begin{aligned} w_s &= \exp(-\mu_s z) \\ w_i &= \cos \mu_i z - \frac{\mu_s}{\mu_i} \sin \mu_i z \\ w_a &= A \cosh kz + B \sinh kz \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

or, when $k = 0$,

$$w_a = C z + D \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\mu^2 = |l^2 - k^2|$, A, B, C and D are constants and the suffixes a, i and s refer to the adiabatic, middle and upper layers respectively.

To evaluate the lee wave number, $k = K$, we assume that w and w' are continuous at the boundaries of the three layers and that $w = 0$ at the ground. Thus the constants A and B can be eliminated to get

$$K \coth (Kh_a) = \mu_i \left(\frac{\mu_i \tan (\mu_i h_i) - \mu_s}{\mu_i + \mu_s \tan (\mu_i h_i)} \right) \quad . \quad . \quad . \quad (6)$$

h being the depth of the indicated layer.

Following Scorer, we can now compute the streamline displacement, ζ, in the airflow past an idealized ridge. The displacement equations are

$$\left.\begin{aligned} \zeta &= H \left(\frac{u_1}{u}\right) \left(\frac{w}{w_1}\right)_{k=0} - 2\pi a^2 \left(\frac{u_1}{u}\right) \left(\frac{w}{\partial w_1 / \partial k}\right)_{k=K} e^{-Kb} \sin Kx \quad x > 0 \\ \zeta &= H \left(\frac{u_1}{u}\right) \left(\frac{w}{w_1}\right)_{k=0} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad x < 0 \end{aligned}\right\} \quad . \quad . \quad (7)$$

where the suffix 1 denotes values at ground level.

Calculation of $\left(\frac{\partial w_1}{\partial k}\right)_{k=K}$ is facilitated by manipulating Eqs. (4) to get

$$\left(\frac{\partial w_1}{\partial k}\right)_{k=K} = \frac{\partial}{\partial K} \left[\frac{\partial}{\partial z} \left(\frac{w_i}{K} \sinh K z_a \right) \right] \quad . \quad . \quad . \quad . \quad (8)$$

and by putting $z_i = -h_i$, $z_a = -h_a$ after the differentiation.

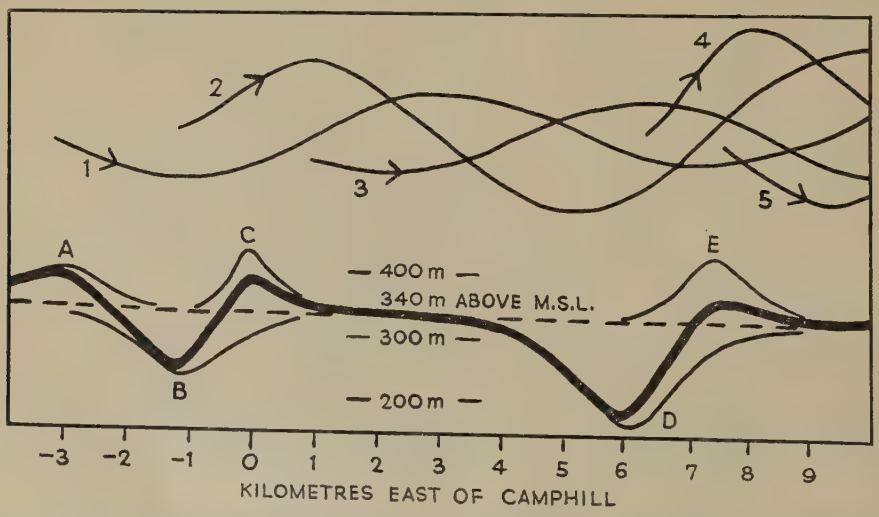

Figure 4. The Camphill topography (thick line) can be obtained by a synthesis of five idealized ridges and valleys (thin curves) superimposed on a datum line (broken line). The ridges and valleys, A, B, C, D and E produce lee waves schematically typified by the streamlines 1, 2, 3, 4 and 5 respectively.

3. The composite flow past Camphill

The thick line in Fig. 4 shows the main topographical features of an approximately east-west cross-section through Camphill. Such a cross-section can also be obtained by a synthesis of three idealized ridges and two valleys superimposed on a datum level of 340 m above mean sea level. This datum level is indicated by the broken line in Fig. 4 and all altitudes now mentioned will refer to heights above this level. Positions and parameters of the idealized ridges and valleys are listed in Table 1 and their profiles are shown in Fig. 4 by the thin curves, A, B, C, D and E. Each of these features gives rise to a set of lee waves, the significant parts of which are typified by the curves 1, 2, 3, 4 and 5 respectively.

TABLE 1. Positions and dimensional parameters of the 'idealized' ridges and valleys used to build up the Camphill topography (all values in km or km²)

Ridge or valley	Horizontal distance of crest or trough east of Camphill	Dimensional parameters a^2	b
A	− 3·2	0·064	0·8
B	− 1·0	− 0·060	0·6
C	0·0	0·040	0·4
D	6·1	− 0·162	0·9
E	7·4	0·050	0·5

Taking $h_a = 1{\cdot}06$ km and $h_i = 1{\cdot}80$ km, the given upper-air conditions accord with a wavelength of 8·3 km which means that, just to the windward of Camphill, waves 1 and 2 reinforce each other to give a pronounced upward component in the airflow. Further to the east the mutual interference between the streamlines serves to illustrate the complex nature of the wave pattern due to a series of ridges and valleys.

A complete picture of the theoretical airflow across Camphill is shown by the streamlines in Fig. 2. This pattern is reasonably consistent with the observed effects. A mean upward wind component of 3·4 m/sec computed for the indicated flight path agrees moderately well with the observed 4·3 m/sec (allowing 1 m/sec for the rate of sink of the aircraft).

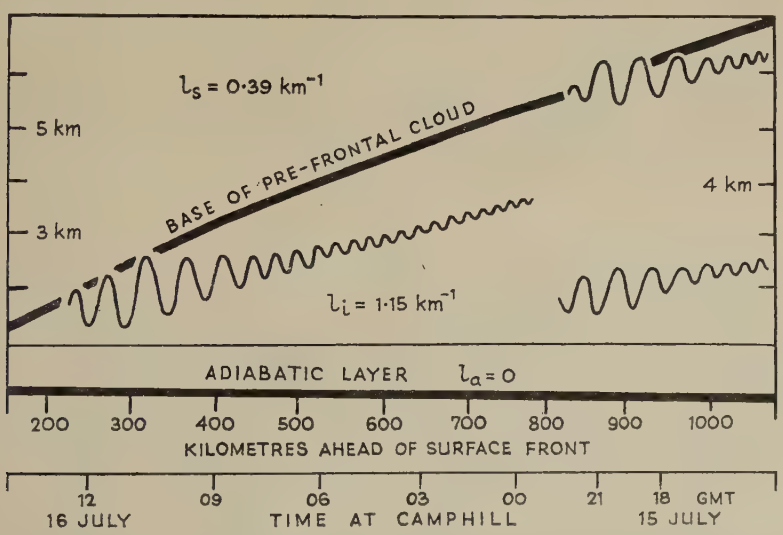
$l_s = 0{\cdot}39$ km^{-1}
5 km
3 km
BASE OF PRE-FRONTAL CLOUD
4 km
$l_i = 1{\cdot}15$ km^{-1}
ADIABATIC LAYER $l_a = 0$
200 300 400 500 600 700 800 900 1000
KILOMETRES AHEAD OF SURFACE FRONT
12 09 06 03 00 21 18 GMT
16 JULY
TIME AT CAMPHILL
15 JULY

Figure 5. Ahead of the well-defined warm front, lee waves are strong in three zones. Their nature, at the levels of their maximum amplitude, is schematically sketched by the curves in these three zones.

4. The nature of lee waves ahead of a warm front

The three-layer characteristic of the troposphere already described is not an uncommon feature of upper-air conditions ahead of a warm front. Therefore, it may be useful to take this front, which moved eastwards across England on 16 July, as a model to show the nature of lee waves likely ahead of a well-defined warm front.

The base of the medium or high cloud is taken to be the warm frontal surface and it is assumed that the three values of l and the depth of the adiabatic layer remain unchanged as the front approaches. Such an assumption is made primarily for convenience but it serves as a basis for discussion and may well be true within about 1,000 km of many warm fronts.

The wavelength now becomes a function of the depth of the stable middle layer which, in turn, is associated with the proximity of the surface front. The relationship between these three items is shown in Fig. 5. In this figure, potential lee waves are illustrated schematically by curves sketched at the levels of their maximum amplitude. Horizontal variation in this amplitude is also indicated by the depth of these curves – but the indication is not drawn to scale.

Some distance ahead of the surface position of the front, lee waves of length 5·4 km are possible, but the amplitude is extremely small. However as the front approaches, the wavelength and amplitude both increase. At 900 km ahead of the front the wavelength measures 10 km and the amplitude is near its maximum; at the 810 km mark the wavelength reaches a limit of 16 km but the waves are again of negligible amplitude. This initial set of waves has maximum amplitudes at two levels, one just above the adiabatic layer and the other just below the frontal surface. A nodal surface exists midway between these two levels.

The continued advance of the waves then brings the effective part of a second set of lee waves which have a single level of maximum amplitude. These waves also increase in length from a little over 5 km to 16 km towards the front. The amplitude of these waves is particularly pronounced between the 220 and 400 km marks and indeed the Camphill wave flights on 16 July were made in this zone.

The present hypothesis indicates that waves in the neighbourhood of Camphill would also have been pronounced between about 14 and 22 GMT on 15 July. Isolated low- and medium-level wave clouds beneath 4/8 cirrostratus were observed there between 18 and 21 GMT. After 2145 GMT it was too dark to make such observations.

5. Qualitative modifications

Usually it is quite impracticable to attempt precise lee-wave prediction on a routine basis. Therefore, the foregoing model may be of some use in forecasting the nature of pre-warm frontal lee waves. Remembering that the parameter l decreases (but not linearly) with stability and varies as the inverse of the appropriate wind component, qualitative modifications of the model can be based on the following theoretical rules :

(i) The lower the value of l_s, the longer the maximum wavelength.

(ii) The higher the value of l_i, the shorter the minimum wavelength.

(iii) The shallower the depth of the adiabatic layer the greater the wave amplitude. The position of the potential lee waves may also be horizontally closer to the front.

6. Turbulence

Observational evidence indicates that, in regions of no vertical wind shear, flight through undistorted lee waves is not turbulent. Waves can, however, produce local increases of whatever wind shear already exists, and it is this increased shear which we may more readily associate with turbulence. Theoretical work on mountain airflow reveals that an increase of wind with height is a favourable, though not dominant, condition for lee waves to occur. Furthermore, the wave amplitude at any particular height depends partly upon the reciprocal of the wind speed at that height. Thus wind shear near the wave crests is increased. Wave troughs can also exhibit increased shear but the mechanism is complex and the increase is likely to be less than that at the crests.

The effect, which has recently been described more rigorously by Scorer (1954), may well be one of the difficult but important factors to consider when assessing the possibility of turbulence in flight. If so, it is apparent that particular attention should be paid to the likelihood of high-level waves well ahead of warm fronts; such waves would accentuate the marked wind shears commonly found near the frontal surfaces or through any jet stream present.

7. Conclusion

Although the observations made were inadequate to subject the theoretical approach to a horough test, they do at least support the computed streamlines. Furthermore, they have uggested a model of warm-front potential lee waves, although, for the present, this should be onsidered as a system to be compared with experience rather than a proven structure. Perhaps uture observations, glider flights and reports of high-level turbulence will confirm or suggest nodifications to the model, but the interpretation of observations calls for considerable care; he composite wave pattern produced by a series of ridges and valleys can vary rapidly as the avelength changes. Consequently, observed wave clouds may change their shape and apparent avelength so quickly and erratically that the true wavelength at any instant is obscure.

Acknowledgment

I am indebted to the Director of the Meteorological Office for permission to publish this aper.

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551.577.2 (41/42); 551.577.3

ome aspects of the variability of annual rainfall over the British Isles for the standard period 1901-30

By S. GREGORY
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(Manuscript received 28 September 1954, in revised form 22 November 1954)

Summary

Although the map of mean annual rainfall in the *Climatological Atlas of the British Isles* is for the period 901-30, those maps which illustrate aspects of the variability of annual rainfall have not been presented for he same period. In this paper, the relative variability for the standard period 1901-30 is mapped and xamined, and compared with the 1881-1915 pattern. This is followed by a brief commentary on the istribution of years of highest and lowest rainfall, the percentage relationship between these extreme values nd the average, and the extreme variability of the annual rainfall, all these also being for the period 1901-30.

1. Introduction

In the *Climatological Atlas of the British Isles*, whereas the mean monthly and annual rainfall listribution maps are for the period 1901-30, the maps of rainfall variability are for the period 881-1915. Even if there were no fundamental differences between the patterns of variability or these two periods, it would nevertheless seem desirable for both mean rainfall and rainfall ariability data to be calculated for the same period, especially as in the original presentation f the annual variability map Glasspoole (1921a) ' emphasizes the importance of remembering hat these maps are constructed on data for a particular period, viz., the thirty-five years 881-1915.'

This present paper concentrates solely on annual conditions, considering not only the elative variability of rainfall for 1901-30 but also various characteristics of the wettest and riest years for the same period. The data were obtained from *British Rainfall*. Only those ecords which were continuous for the thirty years were considered, and of these many were

Figure 1. Relative variability of the annual rainfall of the British Isles, 1901-30 (expressed by the mean deviation as a percentage of the average). Data were taken from *British Rainfall*.

eliminated because of changes in altitude of the gauge during the period. The remaining records were statistically tested for homogeneity, and all doubtful cases were discarded. Finally, a reasonably even scatter of stations being desired, several further stations were not used in the better served areas, leaving a total of 225 records from which to construct the maps. By using these, only three areas are rather inadequately served – north-western Ireland, the Scottish Highlands and the Outer Hebrides – mainly because gauges in these areas were changed in altitude during the period.

Figure 2. Relative variability of the annual rainfall of the British Isles, 1881-1915 (expressed by the mean deviation as a percentage of the average). The map is after *The Climatological Atlas of the British Isles*, p. 79.

2. Relative variability

The pattern of relative variability of the mean annual rainfall for 1901-30, expressed by the mean deviation as a percentage of the average, is shown in Fig. 1. In general terms, variability increased from north to south and from west to east, the largest areas of low values occurring over north-western Scotland and Ireland, and the major areas of maximum variability being over the southern half of England plus central and southern Wales, and over the eastern lowlands and southern uplands of Scotland. The detailed picture indicates that this is not a completely

valid generalization, however. Two of the three areas of maximum values (16 per cent and over) lie within a few miles of areas with values below 12 per cent. Moreover, a broad tongue of low values (below 10 per cent) across Cheshire, SE. Lancashire and northern Staffordshire separates the high values of the Midlands from those of the West Riding, whilst a similar tongue lies athwart the Scottish border between the high values of the southern uplands and those of the Lake District.

A comparison with the 1881-1915 pattern (Fig. 2) shows that not only is there a change in value between the two periods at any specific point, but that the distribution pattern as a whole also presents several differences. Although there is some broad similarity as regards the areas of highest and lowest values, the basically NE.-SW. trend of the isopleths in 1881-1915 is not reflected in the later period. Moreover in 1901-30 there is a greater concentration of high values in the east, especially in East Anglia, Kent and Sussex in the south and along the east of Scotland as far as Buchan in the north.

This does not necessarily invalidate Glasspoole's theory (1921a) of the dual influence of continentality created by Europe as a whole and by Britain as a separate unit. It does suggest, however, that – in so far as one may legitimately speak of continentality in this context of annual rainfall over a considerable period of years – the relative importance of the two may change between one period of years and another. Thus, whereas in 1881-1915 the influence of Britain would seem to have been dominant, reflected by the occurrence of major maximum values inland away from the south and east coasts, in 1901-30 continentality *vis-à-vis* Europe as a whole appears to have been the more important, leading to a concentration of high values in the south and east.

3. Wettest and driest years

The wettest and driest years at individual stations throughout the country have earlier been calculated for the period 1868-1924 (Glasspoole 1926), and it was thought worth while to reconsider this theme for the period 1901-30. The highest annual rainfall totals do not occur in the same year at all British stations. This is equally true of the lowest extremes. Of the 30 years under review, 24 years gave one or other of the extreme values somewhere in the British Isles. 12 years were the wettest somewhere and 15 were the driest, three of the years providing both highest and lowest values in different areas. Thus in 1913, 1915 and 1925, whilst parts of southern England were receiving their highest annual rainfall for the whole of the period, certain areas in Scotland were at the same time experiencing their driest year. The relative importance of the many years involved can be seen in Table 1, where the area of the British Isles receiving extreme values in each year is shown as a percentage of the total area. Moreover the areas which were wettest or driest in the same year are frequently widely scattered throughout the country, e.g., maximum rainfall occurred in 1903 on parts of both the south coast of England and the north coast of Scotland, whilst minimum rainfall occurred in 1902 in some areas of south-western Ireland, on a section of the east coast of Yorkshire, and in the Orkneys.

TABLE 1. Percentage of the British Isles receiving maximum and minimum annual rainfall in any given year during the period 1901-30

Year	Wettest (%)	Driest (%)	Year	Wettest (%)	Driest (%)	Year	Wettest (%)	Driest (%)
1901		1	1911		1	1921		45
1902		18·5	1912	17		1922		2
1903	32		1913	4	0·5	1923	5	
1904		2·5	1914			1924	8	
1905		16·5	1915	0·5	3·5	1925	0·5	0·5
1906		1·5	1916	7		1926		
1907			1917		1	1927	2·5	
1908		0·5	1918			1928	19	
1909			1919		2	1929		4
1910			1920	1		1930	3·5	

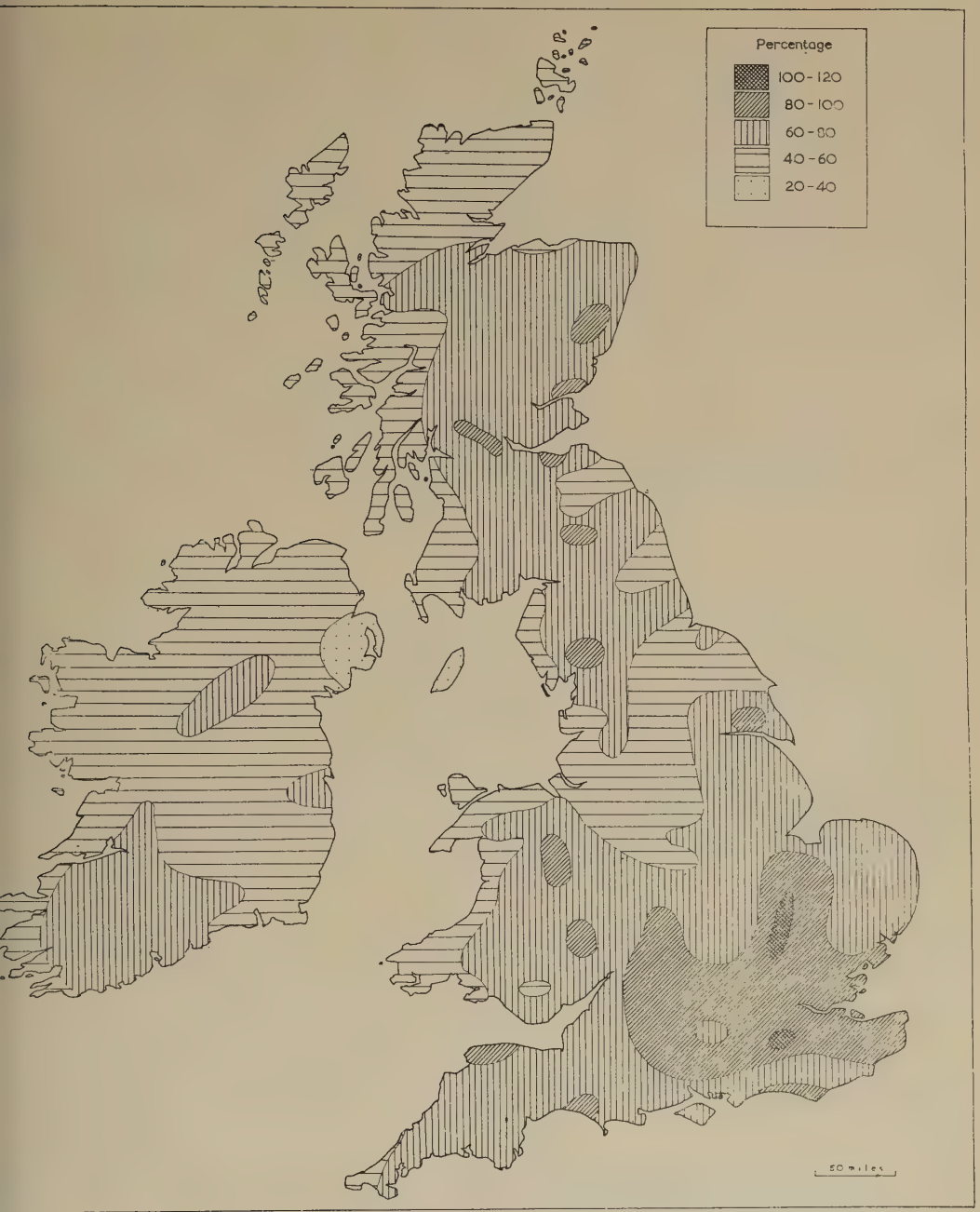

Figure 3. Extreme percentage range of the annual rainfall of the British Isles, 1901-30 (the difference between the highest and lowest annual falls expressed as a percentage of the average). Data were taken from *British Rainfall*.

4. Extremes of annual rainfall

The quantitative relationship between these extreme years and the average value was also considered. From a consideration of the extreme range above the average the country is seen to fall into two parts. Over Scotland, Ireland, the Lake District and the northern Pennines maximum values of 140 per cent or more of the average form a generally north-south belt which rarely reaches the coast, and values decrease outwards from this belt especially towards the

north-west. Over the southern half of the British Isles, the areas of maximum values form an irregular, broken belt from west to east, with lower values especially along western coasts and also over a large part of the Midlands, southern Lancashire, Cheshire and western Yorkshire. This latter central area effectively separates the two belts of higher values in the northern and southern sections of the British Isles.

The deviation of the driest year from the average shows a marked concentration of low values (falls of 60 per cent or less of the average) over southern England, especially in the south-east, with a general decrease north-westwards across the Midlands and Wales. Over the northern half of the British Isles values were relatively uniform (70-80 per cent of the average), with local areas of slightly greater extreme variation, e.g., the east coast and central lowlands of Scotland, and with a change to falls in the driest year of 80-90 per cent of the average in the north-western areas. The contrast between the distribution pattern of the wettest year and that of the driest year is at least partially the result of the exceptional nature of the 1921 drought over southern England (1921b).

Finally, these two values are combined in Fig. 3 to show the extreme range of annual rainfall during the period 1901-30, expressed as a percentage of the average. This map of extreme variability displays a distribution pattern which in many respects resembles that of relative variability (Fig. 1), for eastern and southern areas are again characterized by high values. One significant difference, however, is the location of lowest values over the Isle of Man and south-eastern Northern Ireland rather than over the north-western coasts.

5. Conclusions

The frequent difference of values and distribution patterns between 1901-30 and earlier periods stresses the need for all characteristics of rainfall distribution to be reduced to the same period if full value is to be derived from maps of mean values. This is especially so in the case of relative variability, at least for southern and eastern districts of the British Isles. Some modification of Glasspoole's theory of the influence of continentality is also suggested. The study of the years of extreme conditions stresses the considerable variations in the occurrence and intensity of rainfall throughout the country, especially as only six of the 30 years failed to produce extreme conditions at any of the 225 stations considered. Finally it must be stressed that although the patterns shown are valid for the period 1901-30, there is no reason to presume that they would be equally valid for any other period of years.

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551.501.81 : 551.578.41

Radar evidence for aggregation and orientation of melting snowflakes

By B. J. MASON
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Summary

Measured values of the relative intensities of the echoes received from the melting zone, the snow above, and the rain beneath suggest that, in general, each snowflake in the melting zone coagulates with one or more (on average, two) others of much the same mass to form horizontally-oriented aggregates which scatter as oblate water spheroids of mean diameter/axis ratio 4.

Experimental data

In the simple theory of the radar melting band as first presented by Ryde (1946) it was either stated or implicitly assumed that the flux of particles was the same at all levels, that there were no vertical air currents, that the particles suffered no change of mass after reaching the 0°C level and that the fall velocity of a snowflake did not change appreciably during the melting process but underwent a sudden increase when it finally collapsed into a raindrop. On these assumptions, the theory predicts that the intensity I_m of an echo received by a 3 cm or 10 cm radar from the melting snowflakes would be about five times greater than that I_i from the dry flakes just above the 0°C level, the latter being approximately equal to the intensity I_w of the echo from the rain below the melting zone. Relative values of these three echo intensities as measured by three different groups of workers are shown in Table 1, together with the results of a recent analysis by the present author of data obtained by Browne for a further 12 cases of steady rain. It will be seen that the ratio I_m/I_i is generally considerably greater than 5, while the echo intensity from below the melting zone is often greater than that from above.

TABLE 1. Relative echo intensities from melting zone, snow and rain

Authority	I_m/I_w	I_m/I_i	I_i/I_w
Hooper and Kippax (1950)	5-9	–	–
Austin and Bemis (1950)	2-12 (Av. 7)	Mostly 2-25 (Av. 17) Occasionally > 100	0·1-1·0 (Av. 0·42)
Browne (1952)	4-8	5-15	0·2-1·0
Mason	4-14 (Av. 7) (1 extreme value of 34)	3-31 (Av. 15) (1 extreme value of 93)	0·2-1·6 (Av. 0·8)

There are two factors not appearing in Ryde's theory which may account for these experimental observations : aggregation of the snowflakes in the melting zone, and the fact that scattering cross-sections of the melting flakes will be greater than those of spheres of equal volume because of their flattened shape, and also because once covered with a shell of water they will scatter to all intents and purposes as water particles of the same dimensions (Kerker, Langleben and Gunn 1951).

The purpose of this note is to show that the experimental results of Table 1 can only be explained satisfactorily if the above two factors are taken into account and to deduce the average number of coalescences which occur between wet snowflakes in the melting zone.

2. Interpretation of the data

The ratio of the intensities of echoes from just above and just below the melting zone is given by the following expression :

$$\frac{I_i}{I_w} = \left(\frac{\rho_w}{\rho_i}\right)^2 \left(\frac{K_i}{K_w}\right)^2 \frac{\sum n_i m_i^2}{\sum_{n_w} n_w m_w^2} \simeq \frac{1}{5} \frac{\sum n_i m_i^2}{\sum_{n_w} n_w m_w^2} \quad \text{(for 3 or 10 cm wavelengths)} \qquad (1)$$

where n is the number per unit volume of scattering particles of mass m made of material of density ρ, $K = (\epsilon - 1)/(\epsilon + 2)$, ϵ being the dielectric constant of the scattering material, and the subscripts i and w refer to ice and water respectively.

If we assume that a uniform vertical velocity u prevails throughout the cloud, that at any one level all particles are of the same mass and that each of the snowflakes at the base of the melting zone is composed of ν initially dry flakes, we may write $n_i m_i (v_i - u) = n_w m_w (v_w - u)$, and $m_w = \nu m_i$ so that

$$\frac{I_i}{I_w} = \frac{g_i (v_w - u)}{5\nu (v_i - u)} \qquad (1a)$$

where g_i is a shape factor expressing the scattering cross-section of a dry snowflake relative to

that of a sphere of equal volume, v_i, v_w are respectively the fall velocities of a dry flake and a raindrop relative to the air.

Some recent measurements of the terminal velocities of snowflakes have been made by Langleben (1954), who finds that for dry flakes composed of dendrites the terminal velocity v_i (cm sec^{-1}) is related to the melted diameter d (cm) by $v_i = 160\, d^{0.30}$. For raindrops of diameter D (cm), Spilhaus (1948) gives the approximate relation $v_w = 1420\, D^{0.5}$. Now if ν flakes aggregate to produce ultimately a raindrop of diameter D, then $D = \nu^{\frac{1}{3}}\, d$ and $v_i = 160\, \nu^{-0.1}\, D^{0.3}$. Substituting these relations in Eq. (1a), putting $g_i \simeq 1$ (Atlas, Kerker and Hitschfeld 1953) and neglecting u (of order a few cm/sec), compared with v_i and v_w we have

$$\frac{I_i}{I_w} = \frac{9}{5}\frac{D^{0.2}}{\nu^{0.9}} \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

If no aggregation of the flakes were to occur during melting, i.e. $\nu = 1$, then according to Eq. (1b) the ratio of I_i/I_w would be greater than unity for all values of $D > 0{\cdot}5$ mm. This result is, generally speaking, contrary to observation; the echo from the snow above the melting zone is generally weaker than that from the rain below (see Table 1).

Since most of the measurements were made in steady rain of intensity 0·5 to 3 mm/hr, we may assume a median volume drop diameter $D = 1{\cdot}0$ mm. For values of $I_i/I_w = 1{\cdot}0$, 0·4, 0·2, Eq. (1b) gives $\nu = 1$, 3, 6 respectively. This suggests that on those occasions when values of $I_i/I_w > 1$ are observed, aggregation of melting flakes occurs only rarely; but more usually a flake of mass about 0·2 mg may, on average, coagulate with two or more others during its fall through the melting zone, to produce a raindrop of 1 mm diameter. Putting $I_i/I_w = 0{\cdot}4$, $\nu = 3$, $v_w = 425$ cm sec^{-1} in Eq. (1a) gives $v_i = 70$ cm sec^{-1} – a very reasonable value for dry flakes of mass 0·2 mg.

The ratio of the echo intensity in the melting band to that from the snow above and to that from the rain beneath are :

$$I_m/I_i = 5\, g_m \nu\, (v_i - u)/(v_m - u) \simeq 5\, g_m \nu\, v_i/v_m \quad . \quad . \quad . \quad (2)$$

and

$$I_m/I_w = g_i\, g_m\, (v_w - u)/(v_m - u) \simeq g_m\, v_w/v_m \quad . \quad . \quad . \quad (3)$$

g_m being the shape factor for the melting flakes and v_m their terminal velocity. If we assume the terminal velocity v_m of melting wet snowflakes to be 300 cm sec^{-1} (a value suggested by some observations made both by Langleben (1954) and the present author), for a typical value of $I_m/I_w = 7$, Eq. (3) yields a value of $g_m = 5$. Such a value is consistent with the assumption that snowflakes scatter the radiation as horizontally oriented oblate water spheroids of diameter/axis ratio 4 (see Atlas *et al.* 1953). Substitution of the above values of g_m, $\nu = 3$, v_i and v_m in Eq. (2) leads to $I_m/I_i = 17{\cdot}5$, again a result which accords well with the mean of the measured values.

The usual range of observed values for $I_m/I_w = 4$ to 14 suggests that g_m usually takes values between 3 and 10 (diameter/axis ratios of 2·5 to 7); if the coalescence factor can vary between 1 and 6, then extreme values of the product $g_m \nu$ are 3 and 60. This means that the ratio I_m/I_i may also lie between about 3 and 60, but will only rarely approach the larger value.

Acknowledgment

I am indebted to Dr. I. C. Browne for making his records available and for helping with their evaluation.

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551.594.21 : 523.8

'Stellar thunderstorms'

By C. E. R. Bruce

I have studied Mr. Mason's interesting account (1953) of recent work on the electrification processes in thunderclouds, and should like to suggest a possible extension of the physical investigations, which might clarify these processes, and considerably extend their applicability. In a recent note (1954) I have suggested that electrification will in all probability result from the 'freezing' of materials other than water, and that the electrical discharges resulting from this electrification may account for the phenomena observed in the atmospheres of the long-period variable stars, for example. In these atmospheres, just as in a terrestrial thunderstorm, highly excited atoms suddenly appear in regions of relatively low temperatures.

The materials mainly involved are carbon and the oxides of zirconium and titanium, which solidify at about 3,500°K, 2,500°K and 1,650°K, respectively.

The suggestion is that, as the general atmospheric temperature falls with the star's advance along its evolutionary path, each of these materials in turn will play the major role in a pulsatory process of electrification and breakdown, which would appear to account for the main characteristics of these types of stars.

It would appear to be *a priori* highly probable that other substances besides water will give rise to electrification, either by freezing, or by asymmetrical impacts between the frozen particles, for it is well known that asymmetry in interactions between pieces of the same material can lead to electrification (Henry 1953).

By extending the investigations to cover the freezing *and impact* of other materials besides water, the nature of the processes of electrification in thunderstorms might be more rapidly elucidated.

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The Electrical Research Association,
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6 December 1954.

Discussion meeting on ' Water supply,' 17 November 1954

As already reported on p. 127 of this volume, a discussion meeting on ' Water supply ' took place in the Society's rooms on 17 November 1954. The opening contributions and general discussion are given below.

551.579 : 628

A general survey of the water problem in Great Britain

By D. HALTON THOMSON
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1. Introduction

The function of the water engineer is to find, collect, purify (if necessary) and distribute a public supply, adequate in quantity and under constant pressure sufficient to reach the tops of the highest houses in any given area. The consumer's tap should never run dry.

The consumption of water is steadily rising, and this is the main reason for the present-day development of new sources of supply. A post-war official Committee reported that in the ten-year period 1938 to 1948 the average daily overall demand for domestic and trade purposes in England and Wales had risen from 32 to 40 gallons per head, an increase of 25 per cent, and this increase is undoubtedly continuing.

Water-supply schemes fall into two main types, (1) overground supplies utilizing rivers and streams, the flow of which is, if necessary, impounded in large reservoirs, and (2) underground supplies, whereby water is abstracted by pumping from wells and bore-holes. All these sources depend ultimately on rainfall, the amount and incidence of which is proverbially erratic. Public supply requires that this apparently random element shall be converted into a continuous flow. But only part of the total rainfall is available for supply; a large fraction is lost, partly by direct evaporation and partly by the transpiration of vegetation particularly during the summer months, and from the balance remaining further deductions must be made in practice for economic and other reasons. The net result is that the water engineer has at his disposal for distribution a quantity of water much smaller than the gross amount represented by the rainfall. Apart from obtaining accurate and well-distributed rainfall measurements, the estimation of the various necessary deductions is the major problem in ascertaining the actual quantity available for supply.

In Great Britain overground sources are usual in Scotland and Wales, and in England most of them are in the Lake District, the Pennine Range and Dartmoor; underground sources are mostly confined to eastern, central and southern England.

2. Overground supplies

The obvious method of making the best use of an upland river source is to construct a dam across the valley, forming a large impounding reservoir to store water derived from heavy rainfall for supply during subsequent dry spells and droughts. But, before the dam can be designed, the question arises what is the quantity of water that must be stored to enable water to be drawn off under all conditions at a given continuous rate. This is known as ' equalizing the flow.'

Theoretically, the storage capacity should be large enough, firstly, to retain the whole of the flood waters for future use, and, secondly, to ensure that the reservoir is not emptied before the end of the longest drought. In practice, it is not feasible for economic reasons to construct a dam for a reservoir of this full capacity, because owing to the rarity of the largest floods the reservoir would rarely be completely full, so that the extra cost of providing storage more than some smaller quantity is not justified. Beyond that point the capital cost per million gallons of assured daily supply increases rapidly as the size of the proposed reservoir is increased. It follows that during most winters some water is lost for supply by overflow at the dam.

A consequent problem is, what is the size of the overflow necessary to ensure that all excess flood waters are passed safely away without over-topping the dam. This is vitally important, particularly in the case of earthen embankments. Serious disasters have occurred owing to the lack of an adequate overflow. Clearly, the risk of a cloud-burst over the gathering ground is the prime factor.

A matter of sometimes acute controversy is that of ' compensation water.' No water authority is permitted to impound the whole flow of a stream; a specified proportion of the average flow must be passed through the dam in order to maintain a minimum flow in the natural stream bed on the downward side. This, in principle, is only reasonable, because estate owners, farmers, anglers and water-mill owners have long-standing rights to be respected. The bone of contention is the fair proportion of the available flow to be provided as compensation water; the more for riparian owners, the less for water consumers, and vice versa. The amount is usually fixed by statute before the dam is constructed, and is based on an estimate of the average rainfall less an allowance for evaporation. In the course of years further evidence accumulates, with the result that in some cases the original estimate of the available water resources proves to have been too high, the effect being that the riparian owners are getting too large a share. Reliable rainfall and evaporation records are valuable evidence in support of an application for a revision of such statutory obligations.

All these surface water problems could readily be solved if long stream-flow records were available. Unfortunately, such records are few and far between, because this country is far behind many others in making the matter a State responsibility. A few years before the last war an official Inland Water Survey was set up, and in consequence a limited number of records covering a ten-year period has been published; but on the ground of the national financial stringency the work was suspended in 1952. Fortunately it is about to be revived. The difficulty remains that there are very few records for unutilized upland streams where impounding reservoirs are most likely to be constructed, so that for many years to come it will be necessary to make estimates of surface-water resources indirectly from rainfall records with a suitable allowance for evaporation losses.

3. Underground supplies

In underground water-supply schemes large-scale storage is normally not required, because the cracks and fissures or the granular nature of the water-bearing strata act as a large, if ill-defined, reservoir, which evens out the irregular incidence and amount of percolating rainfall.

Theoretically, natural underground water conditions may be presented in the form of a ' water balance ' over a given period. Thus, on the credit side, at the start of the period so much water is in store, to which is added the total rainfall during the period; on the debit side, so much is lost by evaporation, which includes losses due to the transpiration by vegetation, and so much due to surface run-off. If the amount in store at the end of the period is added to the debit side, the account should balance.

From this account it should be possible to estimate the amount that can be abstracted by pumping for water-supply without any permanent depletion of the water resources over a period of years; there is no harm in a depletion of underground storage over a series of dry years, if it can be shown that this temporary loss will be made good during subsequent wet years. There is, however, one important proviso, namely, the size and capacity of the natural underground reservoir must be known, at least approximately. This is in the domain of the hydro-geologist whose advice is an essential element in solving the problem.

4. Droughts

During severe droughts a situation sometimes arises calling for a most difficult decision. As the drought progresses, the demand for water always tends to increase just at a time when the water-levels in the storage reservoirs and wells are falling, day after day for weeks on end, thus indicating rapidly decreasing reserves. At what stage should the water engineer advise that restrictions on use should be imposed upon consumers ? If too early, he may be accused of undue pessimism; if too late, a serious failure of supply might lead to the shutting-down of local industry and enforced unemployment. How is the risk of a shortage to be measured, and what are the chances that the drought will break by or before a given future date ?

5. Conclusion

From the foregoing it will be clear that engineers are largely interested in extreme values or near-extreme values over periods of varying length, because waterworks must be adequate to cope with such conditions. It is not sufficient to design for the average. But, unlike weather forecasters, they are not concerned with the precise time of occurrence of events – at any rate for the purpose of design; what they require to know is the magnitudes of the extremes that are likely to be experienced during some period in the lifetime of the structure.

But surely, it might be remarked, these problems have already been solved; the operation of existing works must have produced ample data for use in future design. Engineers, of course, have developed many practical rules and formulae based on experience, but they believe that there is room for improvement and feel that advances in science can come to their aid. Waterworks construction involves very heavy financial outlay, and engineers are acutely aware of their duty to see that this is well spent. The desire is to strike the right balance between this expenditure and a reasonable assurance that a water shortage will not occur.

Water engineers are grateful to meteorologists for the collection of data so vital to the solution of their problems. What is wanted is that these data should be presented in a practical form, and that they should be so analysed that the utmost possible information is extracted therefrom for assistance in future design.

It is to be hoped that the papers which follow these observations and the discussion thereon will contribute to these aims.

551.577.3 : 551.579.1/2

Rainfall in relation to water supply

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1. Introduction

Our water supply, whether obtained from rivers, streams and springs, or from underground sources, has its origin in rainfall. Its accurate measurement, as well as a knowledge of its variation in space and time, is therefore of great importance. Initially, consideration will be given to the factors involved in the measurement of rainfall and the interpretation of these observations, and then to the wide variations experienced over the country.

2. Accuracy of rainfall assessments

The accuracy of rainfall assessments used for hydrological purposes depends on the observers, on the instruments used and their exposures, on the distribution of the raingauges over the country and on the scientific interpretation of the observations.

While the collection of reliable rainfall records is one of the functions of the Meteorological Office, the observations have been supplied both by private individuals, who maintain records for their own interest, and authorities engaged in various public services, especially those concerned with some aspect of hydrology. This combination, of the interest of the observer in the local rainfall and the collection of the results by a central authority, promotes accuracy and uniformity in procedure. The recent establishment of River Boards in England and Wales provides an opportunity of securing stations in areas so far unrepresented and also of ensuring greater permanency in the records. The periodic inspection of the contributing stations, which now exceed 5,000, is essential, but, unfortunately, it has not been possible, during recent years, to inspect as many stations as planned.

The standard equipment and the procedure for measurement are described in *Rules for Rainfall Observers* (obtainable by observers on request from the Meteorological Office, Harrow, Middlesex). These rules are based on experience gained since the collection of rainfall data commenced in this country in 1860. Certified instruments should be used and the gauge tested for leaks each year, especially after periods of frost. The rim should be set one foot above

ground. At higher levels there is a loss, due to wind eddies set up round the gauge, which on the average is 1 per cent for each foot above one foot. If the rim is less than one foot from the ground, or surrounding surface, then insplashing may occur. Special precautions are necessary in the selection of a site for a raingauge, which must neither be too sheltered nor too exposed. In over-sheltered sites the raingauge catches a fair sample of the rain which falls there, but the catch is representative of too limited an area, some adjacent area receiving the rain which is obstructed. In over-exposed sites the raingauge fails to catch a fair sample of the rain which falls there. Loss due to over-shelter is much greater in windy localities than in town sites, where the force of the wind is already broken by surrounding buildings. Thus a gauge in the courtyard of the Victoria and Albert Museum gave only 3 per cent less than that recorded in adjacent orthodox park sites, although the buildings surrounding the gauge were 60 to 90 ft high and only 50 to 90 ft away. A gauge on exposed moorland catches less than the proper amount, owing to wind-eddies set up round the gauge, just as when the rim is raised more than 1 ft above the ground. Gold (1928) has stated that ' the exposure of a raingauge may be considered as sufficiently near the normal to require no artificial screening if the wind at the level of the rim of the gauge does not exceed 10 to 15 mi/hr.' Further details on the proper exposure of a raingauge are set out in the ' Report on the determination of the general rainfall over any area,' prepared by representatives of the Royal Meteorological Society, Institution of Water Engineers (1937) and the British Rainfall Organization (Meteorological Office). Much still remains to be learnt about the effect of wind on the catch of a raingauge and further study of the records is required, especially of the annual and average values, in relation to the differing exposures of the raingauges set up in mountainous districts. A precise measure of the effect of an abnormal exposure can often be ascertained by examining the values on a map, or by arranging for overlapping records at a nearby orthodox exposure. Many investigations have been carried out to define the best exposure for raingauges and that published by Hudleston (1933) warrants close study.

Having obtained as reliable readings as possible from competent observers with standard equipment some critical examination of the large mass of figures is still required. It is desirable that any queries which may arise should be referred to the observer while the facts can still be recalled. Many water undertakings examine their records in detail before forwarding them to the Meteorological Office, and this arrangement is recommended. In the Meteorological Office the records are scrutinized and daily values compared, so that any accumulations or omissions may be rectified. The cartographical examination of the monthly and annual values also brings out inconsistencies which can be cleared up by examination of the daily values, or by consulting the observer. On a map of annual values an error of 5 or even 10 per cent at one station may easily be overlooked because of the wide local variations. If, however, this error persists for two or three years, then it should become apparent. The error can be more readily discerned through an inspection of the station by one experienced in the various types of error involved in rainfall recording.

The greatest uncertainty in the readings occurs with snow, but fortunately in this country snow contributes only a small part of the total precipitation. To secure reliable daily and monthly readings during periods of snow it is important to follow the procedure set out in *Rules for Rainfall Observers*, Section 11. This involves :

(i) Taking special precautions at each daily gauge to record (*a*) the amount of water in the gauge, (*b*) the amount of unmelted snow in the funnel after conversion to water and (*c*) the depth of fresh undrifted snow.
(ii) Taking additional readings of the depth of fresh undrifted snow in the locality.
(iii) Arranging for interim readings of monthly gauges, especially when the thaw sets in, in order to limit as far as possible the period of uncertain readings.
(iv) Critically examining the final values on maps, not only for the month but for periods covering the duration of the snowy weather.

It has been found that the over-estimation of snow through drifting at some stations is about equally offset by the under-estimation of snow at other stations where snow is blown out of the gauge. Nevertheless further investigations are required to secure greater accuracy of measurement, to record the snowfall in the uninhabited mountainous regions and to distinguish between snow which reaches the ground direct and that which falls on exposed summits, etc., and later is blown off into valleys below.

The water engineer is mainly concerned with the general rainfall over the area draining to his reservoir. Estimates of the mean rainfall over an area present difficulties because the stations are not uniformly distributed, the mountainous or wetter regions being usually less well repre-

sented by raingauge sites. Moreover, because of the amount of work involved in plotting the values, drawing the maps, and measuring the zones of differing rainfall limits with a planimeter, short cuts or approximations sometimes need to be made and these result in less precision in the final values. With the increasing demand for water, greater precision is required in estimates of general rainfall in order that the fullest use may be made of this national asset. Moreover, as more run-off data become available and estimates of evaporation are made, more precision in the rainfall estimates will be required, in order that the whole hydrological cycle may be studied.

3. Variation of annual rainfall over the country

In considering the available water supply from any area an estimate of the average annual rainfall is required. In order to compare the potentialities of one area with those of another it is important to use the same period. It is found that homogeneous records are rarely available for periods longer than 30 to 40 years, and as a matter of convenience the period 1881-1915 has been used as the standard of reference for many years. There will eventually be a change to the more recent period 1916-1950, so that averages for two consecutive 35-year periods will be available, although generally for different stations in each period. The methods employed for working out averages are set out in an article on the ' Rainfall of Norfolk ' Glasspoole (1928). It has already been explained that the records vary in their reliability; they cover different periods and they may not be homogeneous. The problem is, therefore, that of making the best use of the available information by analytical, cartographical and statistical methods.

The computation of station averages is important because it gives a very much closer network of values on the map than is available for any individual year. This enables the records to be examined critically and the distribution defined in greater detail. As explained earlier, errors up to 10 per cent may be undetected on the annual map, but on the average map persistent errors, such as those due to exposure, become quite apparent if they are even as small as 1 per cent. Thus over the County of London, with an average of 24 in., station averages fitted well enough to draw isohyetal lines at $\frac{1}{2}$ in. intervals, so that any station with an average in error by 1 per cent was noticeable. This accuracy was only attained as a result of inspecting the stations, replacing gauges on roofs by standard gauges in parks and waiting until records were available for a long enough period to compute averages. The closer network of values and this critical examination of the records justifies the average distribution being defined on maps on a scale of 2 mi/in.

The distribution is naturally most uncertain in regions of fewest records and over the mountains, where the increase in rainfall with altitude needs to be judged from few records and analogy with other areas. An advantage of considering the average distribution is that it is primarily dependent upon the orography, because of the effect of the configuration in producing rising air. There is considerable scope for determining the maximum rainfall likely to occur on mountain tops in various parts of the country, either by direct measurement, from theoretical considerations or by the use of wind-tunnel experiments. Indeed a physical analysis of the increase of rainfall with altitude is overdue, especially in view of the difficulties of determining the rainfall in mountainous regions by direct measurement.

A general picture of the distribution of average annual rainfall over the country is given in the maps of average annual rainfall covering Great Britain on a scale of 10 mi/in. published by the Ordnance Survey (1949), as prepared in the Meteorological Office. Where the area is to be used for a water supply or hydro-electric scheme it is essential that all the data within the region should be reviewed, since in most cases a more detailed distribution can be given than is possible on this generalized map, especially in mountainous regions. This should involve the inspection of the existing raingauge sites and an assessment of their reliability on the spot, where the lie of the land can be more appreciated than by examination of a contoured map.

As a first approximation the average annual rainfall is a characteristic of each locality. The average increases from the plains of the east, with a minimum of 20 in. around the Thames Estuary, to the mountainous regions of the west, with more than 100 in. in parts of North Wales, the English lake District, the southern uplands and the west of Scotland, reaching a maximum of 200 in. at the summit of Snowdon and at the head of the River Garry in Invernesshire. From the aspect of water supply it should be noted that where the rocks are permeable and the supplies mainly underground the average annual rainfall rarely exceeds 30 in., while where the rainfall is heavy the rocks are mainly impermeable and more suited to the construction of storage reservoirs.

The average annual rainfall is also used in estimating monthly rainfall over specified areas r comparison with run-off data. Theoretically the monthly rainfall over an area is obtained y plotting all the values, drawing the isohyetal lines, measuring the area between these lines ith a planimeter, estimating by eye the mean rainfall of the area between these lines, multiplying e mean rainfall by the area concerned, and dividing the sum by the whole area to give an timate of the general rainfall. This method serves well for an area, like the Thames Valley, here there are many records and also where there is little variation in the rainfall over the area. a mountainous area with few records available it is often better to express the monthly rainfall each station as a percentage of the station annual average, estimate the average general rainfall ver the area and apply the mean monthly percentage based on representative stations to this nnual average for the area. In this way it is possible to use all the details on the average annual ap which may be based on many records maintained for earlier years. The procedure is xplained in the joint Report of 1937, referred to in § 2 (Institution of Water Engineers 1937).

4. Variability of rainfall with time

In this country a rough generalization is that the rainfall of the driest year is two-thirds f the average, that of the driest two consecutive years three-quarters and of the driest three onsecutive years four-fifths of the average. The definition of the frequency of such extremes s of importance to the water engineer both in the design of impounding reservoirs and in lanning for the maintenance of supplies. Many statistical investigations have therefore been arried out to define the fluctuations of rainfall – see Glasspoole (1921, 1922 and 1925).

With annual values the ratio of the mean to the standard deviation closely approximates t each station to the normal value of $(\pi/2)^{\frac{1}{2}}$ or 1·25. The mean deviation varies from 8 per cent f the annual average along the west coast of Scotland and in Northern Ireland, to 15 per cent in entral England, from Gloucester to Derby. The percentage mean deviations are remarkably milar month by month and approximate closely at each station to the normal values of $(12)^{\frac{1}{2}}$ mes that for the year. In planning for the future, this greater uniformity of the rainfall in he north-west and the greater variability in central and south-eastern districts needs to be ken into account. It may be that there is some difference in the variability of the percentage ainfall at valley and nearby mountain stations, but sufficiently homogeneous records are not et available to define these relatively small variations with any precision. Nevertheless, from knowledge of the average annual rainfall and the standard deviation the frequencies of annual nd monthly rainfall can be given for any part of the country as a first approximation.

The frequency of long periods of no measurable rain is not so easily defined by considerations f the annual and monthly rainfall and requires separate study. The distribution of the greatest umber of consecutive days with no measurable rain does, however, reflect that of the annual mounts and of the mean deviation. Thus no absolute droughts (at least 15 consecutive days ithout measurable rain) lasting as long as 20 days are on record (since 1887) in parts of he extreme north of Scotland and north-west of Ireland; over most of the country the maximum uration is between 30 and 40 days; absolute droughts of 40 days or more have occurred only cally in eastern, central and south-eastern England; the largest area with more than 40 days ccurred over south-east England from the Isle of Wight to the Thames Estuary. An absolute rought occurs on the average once a year in the south-east of England from Somerset and loucester to Suffolk and Kent but only once in five years over much of the north-western half f Scotland.

The seasonal distribution of rainfall is of importance because during the summer half-year uch of the rainfall is lost by evaporation or absorbed by vegetation. On the average more in falls in the winter than in the summer half-year everywhere, except in limited areas in the st. Much of the south-west of England and Wales and of the Western Highlands of Scotland ceives as much as 60 per cent of the annual total during the winter half-year (Glasspoole 1928). his emphasizes the importance of the storage of winter rains against summer droughts, losses y evaporation and the requirements of agriculture.

So far attention has been directed mainly to the rainfall of the standard period 1881-1915, ut consideration needs to be given to the general trends over a longer period. Looking at the cords for England as a whole, it is apparent that wet years have alternated with dry years in irregular manner. This frequent change and the occurrence of very dry years amongst wet ars is often unfortunate for the promotion of new supply schemes. Before plans can be approved e general public has often recovered from the effects of a drought and tends to treat lightly

the risks of similar dry years in the future. Superimposed on this relatively quick change fro dry to wet years it is apparent that there are longer periods in which dry or wet years have pr dominated. The general trend may be summarized as below; the number of years being includ within brackets :

Dry years predominated	Wet years predominated
1941-1953 (13)	1909-1940 (32)
1887-1908 (22)	1872-1886 (15)
1853-1871 (19)	1816-1852 (37)

The mean rainfall of the 13 years 1941-1953 was just less than the 1881-1915 average, whereas the previous 32 years 1909-1940 gave 6 per cent more than the 1881-1915 average, so that there has been a general decrease in the rainfall of recent years. Another striking feature is that a run of four consecutive dry years has not occurred since 1887-1890 (with a mean of 89 per cent) and of five consecutive dry years since 1854 to 1858 (with an even smaller mean of 87 per cent of the 1881-1915 average). The recurrence of runs of such generally dry years should therefore not be overlooked.

One of the objects of the preparation of these general values over England covering a long period was their possible use for indicating the future trend. Statistical examination of this long series of rainfall records has shown, however, that while the general trends in past records can be expressed mathematically, the resulting equations cannot be used to indicate future trends. The reason for this is that old periodicities drop out and new ones appear and the causes of these changes is not yet understood. Some attempt has also been made to understand the causes of droughts in this country by studying the associated pressure distributions over the northern hemisphere. Again, progress has been limited, owing to the complexity of the factors involved and the dearth of information when viewed on a world basis. The results of both these investigations are set out in *British Floods and Droughts* (Brooks and Glasspoole 1934). The water engineer and the general public would clearly benefit from any progress which could be made from either of these aspects.

5. Reliability of rainfall

Although endeavours to unravel the trends in past records, in order to predict those of the future, have so far been unsuccessful, considerable progress has been made in defining the frequency of occurrence of rains of specified amounts to give an indication of those likely to occur in the future.

Examination of long records of 70 years or more, maintained at a number of well-distributed stations over the British Isles, has brought out two important points (Glasspoole 1930, 1947 and 1951). The extremes and frequencies of amounts in groups of consecutive months are very similar at all stations when the amounts are expressed as percentages of the average annual rainfall for the station concerned. Further, the bulk of the percentage amounts in groups of consecutive months are confined to narrow limits. Indeed, while the weather of the British Isles is remarkable for its variation from day to day, this variability naturally results in a rainfall which is strikingly uniform and reliable when judged from monthly and annual totals.

Thus a reasonable expectancy is that even in the driest years any group of four consecutive months will give at least 9 per cent of the average annual rainfall at any station. Further, if any station has recorded this minimum of 9 per cent in four months, then the next four months can be reasonably relied upon to give at least 21 per cent making 30 per cent, for the eight months. This is an important consideration in planning to maintain our water supplies.

The reliability of rainfall in this country may be illustrated by quoting figures for four consecutive months. At all stations in the British Isles the entries for four consecutive months can be regarded as varying between 9 and 77 per cent of the annual average, but for four-fifths of the entries the range is 19-51, for three-fifths 22-45, while one-fifth of the entries are confined to 28-36 per cent of the annual average.

While the fluctuations are similar at all stations as a first approximation, it is clear that the other variables over the country, of standard deviation and seasonal distribution, need also to be taken into account in defining the fluctuations with greater precision. There is therefore considerable scope for applying modern statistical methods to rainfall data in their application to hydrological problems.

6. Heavy falls on rainfall days

The reliability of the rainfall of the British Isles results from the frequent changes in the weather and the prospects of heavy rains at all times of the year. At most stations one-quarter of the annual total is contributed by about the 10 wettest (not necessarily consecutive) days, half the annual amount by the 30 wettest days and three-quarters by the 70 wettest days. Days with small amounts of rain produce little run-off because of losses by evaporation, so that the water resources of this country are dependent on the rainfall of a relatively few days during the year. A study of the incidence of these wet days is therefore likely to be a profitable line of investigation in considering water resources.

The areas covered by intense and widespread falls of rain need also to be considered in taking precautions to prevent reservoir embankments being swept away and in determining the length of weir necessary to cope with the overflow during periods of flood. All outstanding rains have been studied in the annual volumes of *British Rainfall* and the results summarized to define the variations in amount, duration and areas covered by rains of varying amounts (Glasspoole 1930). While the previous sections of this paper have indicated that much of the fluctuation experienced in this country could have been deduced from relatively few homogeneous records covering long periods, intense falls vary so much within short distances that a close network of stations, equipped with recording gauges, is essential to define with precision the variation and incidence of the rainfall during such storms. Here the meteorologist can assist the water-supply engineer but the problem has a much wider application, including land drainage and the drainage by sewers of large towns.

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551.579 : 628.1

Water supply

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1. Introduction

The subject of water supply covers a wide field, but the present discussion has been narrowed to consider the assessment of yield of water from various types of sources from the point of view of the engineer concerned with public water supply.

The methods which have been adopted for calculating the amount of water which can be obtained from a selected source will be described. Examination of these methods will show the need for additional hydrological data for the more precise methods required, as undeveloped water resources become fewer.

Public water-supply projects can only use a proportion of the nation's water resources and it can be a difficult problem to reconcile their needs with other interests such as industry, agriculture, navigation, fishing and amenity (including mitigation of pollution).

Water schemes cost large sums of money, but capital expenditures on the development and maintenance of industries and town communities depend on the provision of adequate piped water supplies. All the more reason, therefore, for spending a comparatively small sum on investigations which will facilitate the preparation and successful completion of future schemes.

2. Estimation of requirement

Before preparing a project to obtain additional water resources for a community, an assessment must be made of the quantity of water required. The assessment will be made for a particular year in the future for which it is considered reasonable. A period of 20 to 30 years ahead is commonly selected.

Not only must the yield of a selected future source be determined with accuracy, but the water available from the existing works must also be known. Errors in the estimations may lead to inadequancy of the new works or excessive expenditure of money on unneeded supplies.

The importance of accurate estimation is indicated by Table 1.

TABLE 1. Capital costs of various recent water-supply projects

Type of source	Approximate cost £ (thousands)	Quantity of water (10^6 gall/day)
Direct intake	650	8
Direct intake	1,660	6
Borehole and rural scheme	120	0·35
Impounding scheme	200	1·25

Having determined the amount of water wanted, the engineer will examine the sources from which it can conveniently be obtained, bearing in mind :

(a) Locality in relation to the authority to be served, (b) quantity available noting any chance of further extensions at minimum expense, (c) the quality of the water and the cost of treatment, (d) time required for construction, (e) the importance of possible opposition to the project involving a consideration of the other interests in the same water resources, which can be many.

The source finally selected will generally fall into one of three major categories :

(1) Direct abstraction of surface water from a watercourse.
(2) Surface water impounded in a large reservoir.
(3) Underground water.

It is proposed, therefore, to consider the methods in use for estimating the productive yields of these various types of water-supply project.

3. Assessment of yield from an unreservoired surface catchment

Owing to the wide fluctuations in the daily flow of a river, the amount of water which can be abstracted is limited by the dry-weather flow and the amount which must be left in the river for other purposes. If storage capacity is available then powers might be obtained to abstract larger amounts when the flow in the river is greater, but if pumping plant or pipeline capacity is involved, the amount which can be taken in that way is limited.

Fig. 1 showing a typical duration curve for the flow of a river in a particular year indicates the small proportion of the total annual flow of a stream which can be usefully abstracted for supply owing to the large proportion of the annual discharge occurring on a few days in the year.

The main problem, in such a case, is to determine the lowest flow reasonably likely to occur once in say twenty or fifty years according to the risk that can be taken. It is also important to know the lowest average flows over periods of 7, 14 and 28 consecutive days, and the shape of the lower part of the duration curve during a dry year.

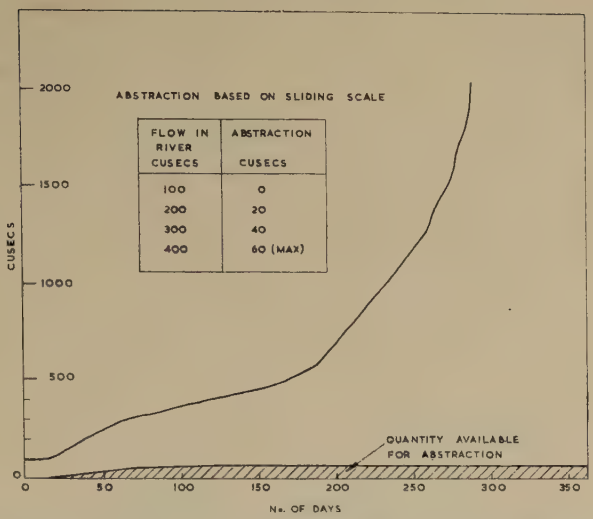

Figure 1. Duration curve river ' X,' 1946-7, indicating the number of days when the flow is equal to or less than a specified amount (1,000 cusecs = 539 × 10^6 gall/day).

Unfortunately this ideal information is very rarely available, and must be replaced with estimates obtained by relating data from another catchment, or empirically from knowledge of the catchment area, its extent, topography and geology.

4. Assessment of yield of a reservoired surface catchment

In order to make more complete use (for water-supply purposes) of the flow in a stream or river, it is necessary to provide large storage reservoirs and where this storage is provided by forming a reservoir in the stream valley by a dam, the stream is said to be ' impounded.'

In the second half of the 19th century, when the importance of pure water supplies came to be realized, the large towns started to seek new supplies from impounded streams in the sparsely-inhabited hill regions such as Longdendale for Manchester, Rivington and Vyrnwy for Liverpool and Elan for Birmingham.

The engineers responsible for these schemes had to determine the economic storage capacity which could be provided in the impounding reservoirs and various rules, based empirically on rainfall records, were developed. The amount of water to be considered as available was based on a proportion (four-fifths) of the annual average rainfall from which was deducted the amounts lost by evaporation and required to be discharged as compensation water.

Bearing in mind the errors likely to arise in measuring rainfall in upland areas and the difficulty in making allowance for loss by evaporation, it is not surprising that there are occasional discrepancies between the originally estimated yield of a source and the yield later found to be actually available.

In fact one is more surprised at the degree of accuracy which the pioneer engineers did attain.

It may be many years after the construction of an impounding reservoir before it is severely tested by drought conditions and errors of estimate brought to light. The dry spell of 1933-4 caused a great deal of anxiety and as a result the Institution of Water Engineers Hydrological Committee prepared a report in 1936 putting forward a standard basis for estimation of yield. The report was severely criticized when it came up for discussion, and as a result it was never accepted and published.

All these earlier methods were based on rainfall and suffered from the consequent uncertainties. Logically, records of stream flow from the catchment to be reservoired should form the basis of design, so eliminating the uncertainties of rainfall measurement and evaporation loss, and also ensuring that the effects of ground-water storage in maintaining dry-weather flow are included.

Although records of stream flow in this country are still hopelessly inadequate, there have been attempts in recent years to relate the run-off of a catchment and the storage capacity of the reservoir required.

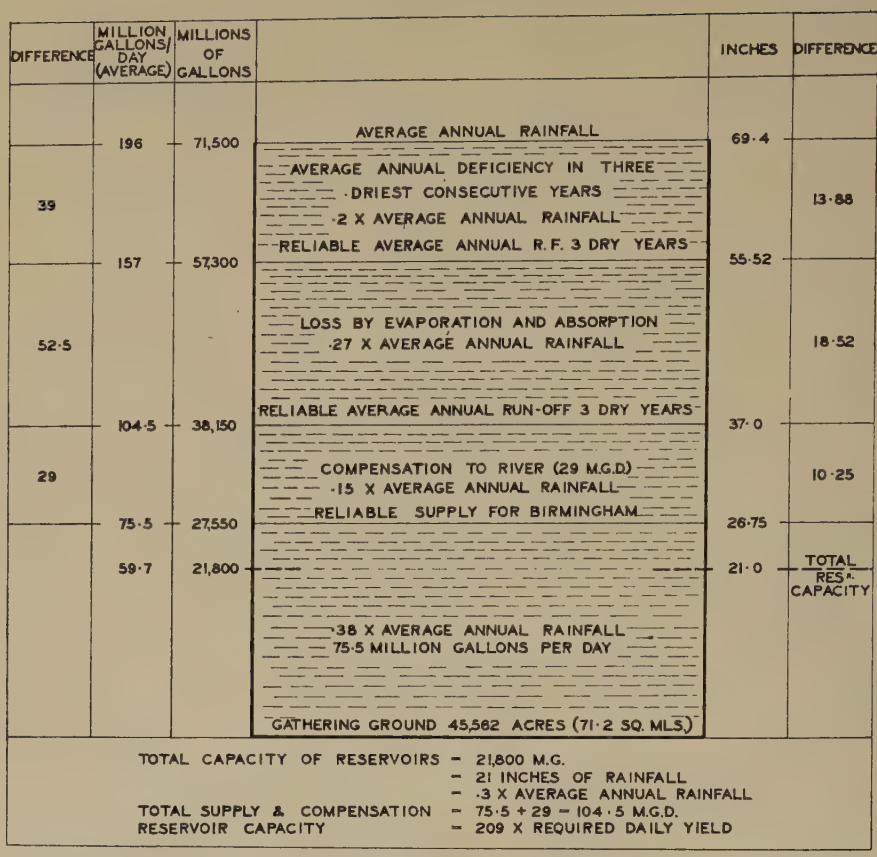

Figure 2. Rainfall on Elan and Claerwen gathering grounds.

A diagram constructed by Lapworth (1949) shows the relationship between annual run-off, storage capacity and yield of a reservoir. Thompson (1950) indicated a statistical method of analysing run-off data using moving averages of total run-off in various time periods and applied it to the 43-year record for the River Derwent showing that the 1933-4 drought provided a yield almost equal to the value to be expected statistically once in 100 years with a 1 in 10 chance of a lower value in any given period of 100 years. Law (1953) has developed a correlation method applicable where long-period rainfall and short-period run-off data are available to produce a cumulative deficiency diagram which can indicate storage requirements with either uniform or non-uniform rates of draw-off. No estimate of evaporation loss is required in this method.

Where records of run-off are available and include an exceptionally dry period of sufficient duration, a cumulative run-off diagram represents a simple method of determining the storage required to equalize the run-off over that selected period. The example in Fig. 3 shows the storages (600×10^6 and 950×10^6 gall) required to provide uniform flows of 4×10^6 and $4{\cdot}6 \times 10^6$ gall/day equalized over 2 years and 4 years respectively.

Judgement is required to relate the severity of drought in the period studied to that which must be provided for.

A variation of the cumulative run-off diagram prepared for a selected period is the minimum run-off diagram based on selections from the whole record of run-off of the driest, 1, 2, 3, 4 etc., consecutive months and their amalgamation to form a curve of probable minimum run-off.

It is apparent that with inadequate long-term stream-flow records in the British Isles, the methods for designing the capacity of impounding works must be varied to suit the particular circumstances and information available. Recommendations were put forward for discussion in 1953 by the Hydrological Research Group of the Institution of Water Engineers (1953).

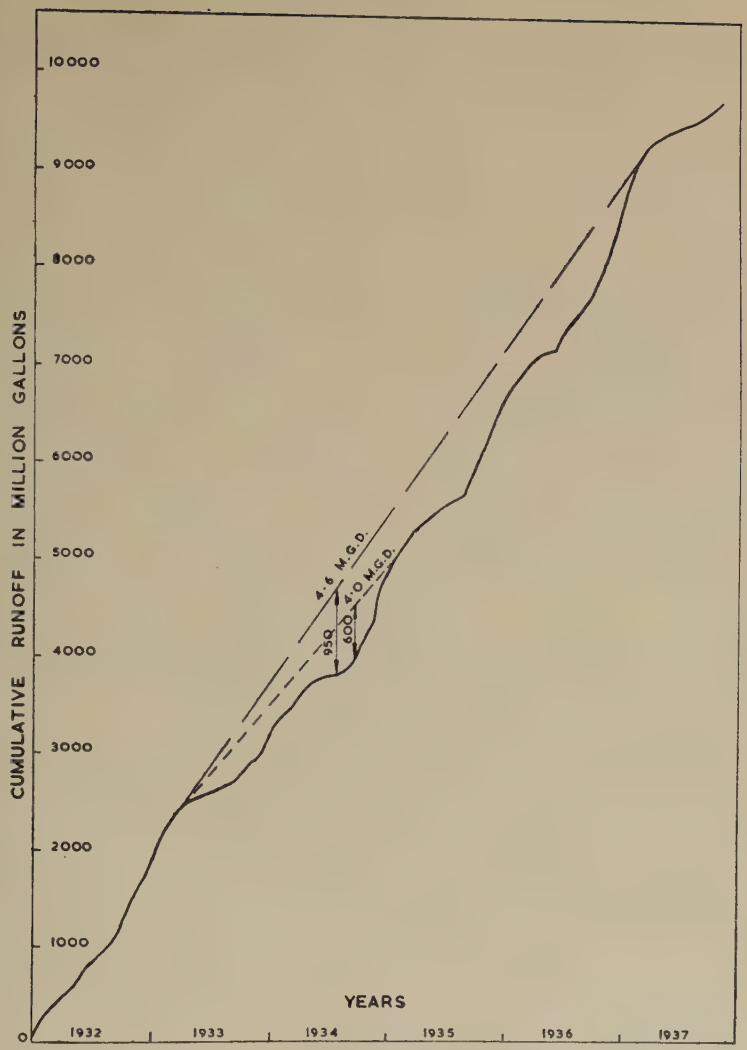

Figure 3. Cumulative run-off diagram.

Obviously, once impounding works have been constructed on a river, careful records of infall and run-off should be maintained in order to check and modify, as experience demands, e estimates originally made.

It is in this process of checking that statistical methods, particularly correlation, may prove ost helpful.

5. Assessment of yield of underground resources

The quantity of water to be obtained from a well or borehole or a group of wells or boreholes, pends on two main assessments.

(a) Assessment of the extent of a water-bearing stratum or aquifer and of the area exposed absorb water, and the determination of the amount of water which can be drawn from the uifer without so disturbing the hydrological conditions as to affect the reliability of the supply seriously to injure existing uses of water from the aquifer.

(b) Assessment of the adequacy of the well or group of wells to obtain the quantity of water ailable and required.

In order to appreciate the problems of assessment (a), i.e., the yield of the whole aquifer, is necessary to consider the water-balance equation, together with the sub-components of the ain factors (Fig. 4).

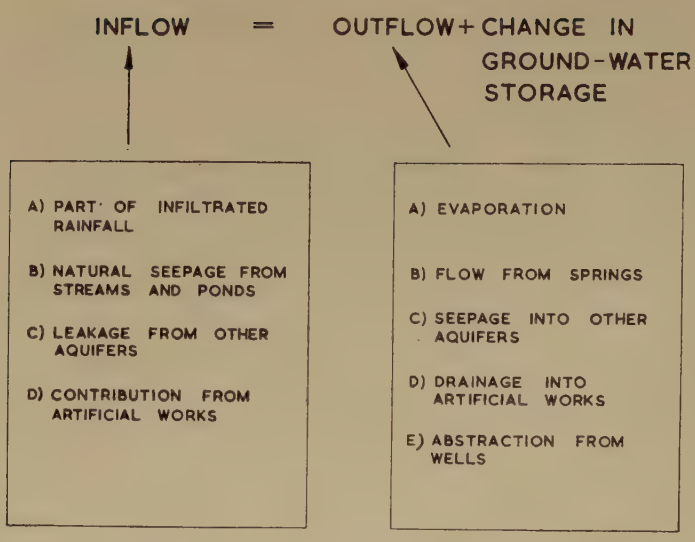
INFLOW
=
OUTFLOW + CHANGE IN GROUND-WATER STORAGE
A) PART OF INFILTRATED RAINFALL
B) NATURAL SEEPAGE FROM STREAMS AND PONDS
C) LEAKAGE FROM OTHER AQUIFERS
D) CONTRIBUTION FROM ARTIFICIAL WORKS
A) EVAPORATION
B) FLOW FROM SPRINGS
C) SEEPAGE INTO OTHER AQUIFERS
D) DRAINAGE INTO ARTIFICIAL WORKS
E) ABSTRACTION FROM WELLS

Figure 4. Ground-water storage equation.

It will be noted that the majority of the sub-components of the above equation are based measurements for which few data are available, and in some cases the direct measurements a at present an impossibility.

The general method currently used to assess the water potentialities of an aquifer is determine as closely as possible from a geological survey the catchment area through which ra reaches the aquifer, and to assume a proportion of the average annual rainfall as ultima percolation.

Allowances, depending on judgement and experience, need to be made for surface conditio of the catchment, losses from the aquifer by spring flow, etc., variations in incidence of rainf and other hydrological influences.

When it is remembered that the quantity of water left to be abstracted is only a margin quantity remaining after the subtraction of one or more larger values from the total rainfa on the catchment, an error equivalent to 1 in. of rain in any of the major assumptions can b sufficient to determine the fate of a project costing large sums of money and affecting the interes of large communities.

As the quantity of water being drawn from an aquifer approaches the limit, so the preci determination of such limits becomes more important, and the fact that such a situation is arisin over large areas of the country is demonstrated by the extent to which the Minister of Housin and Local Government has scheduled districts in which no wells can be sunk more than 50 deep without his consent.

The assessment of the second factor, namely the ability of a well or group of wells to explo the available water resources of an aquifer, involves knowledge of the geological characteristi determining the rate at which water will flow through the aquifer. This is a factor of great varia bility over a small section of the same aquifer and is one of the reasons for the test-pumpin procedure which should be applied to every well before it is brought into commission.

6. The factors affecting the yield of water resources

Having briefly examined the methods for determining the yields of our various water-suppl resources, we should consider the factors on which calculations are, or should be, made an the present state of our knowledge of those factors.

1. *Topography.* The excellent maps of the Ordnance Survey provide an adequate basis t which other special information can be added.
2. *Geology.* The geological maps of the country were adequate, but the delay in making man of the sheets readily available since the last war has been a handicap.

3. *Rainfall.* Thanks to the efforts of early workers and their successors, we have records which tend to hide the great need for other hydrological data. There is a fine record for the continuous publication of rainfall data and the chapter on seasonal rainfall included in *British Rainfall* is useful.

   However, the emphasis on the average of the calendar year may be misleading and this year is an example. The ground water stored is largely dependent on the winter rainfall and there would seem justification for publishing the winter and summer figures for the water year October to September.

4. *River-flow records.* The lack of published records of the flows of British rivers has been the subject of expressed regret on many occasions in past years, and the suspension of the work of the Inland Water Survey nearly three years ago led to many protests. The recent relaxation of this suspension does not by any means solve the problem, as such few records as were available and published before the war have not appeared since. To all intents and purposes the comprehensive Inland Water Survey of Stream flows has yet to start, and even now we are not promised a regular published record.

5. *Evaporation and percolation.* Apart from the few records appearing in *British Rainfall* and the publications of individual enthusiasts, little has been done on the collection and publication of records – in fact there are few records suitable for collection.

   Hydrology can be affected by a number of other meteorological factors such as wind, temperature and hours of sunshine which apparently are measured regularly in many places, but presentation in a form easily suitable for hydrological calculation is an essential to a study of the extent of their effects.

   Another subject affecting hydrology of which little is known at present is the effect, if any, of vegetation on rainfall, evaporation, percolation and rate of run-off. With the rapid extension of afforestation in upland areas, the question whether trees, particularly conifers, affect the total run-off and percolation of water should be answered as soon as possible.

7. General matters of meteorology of interest to water engineers

So far no mention of floods has been made in this communication and lack of time prevents its proper discussion. Suffice it to say that the water engineer has grave responsibilities in designing flood weirs and by-wash channels on large dams and in the control of rivers. The lack of quantitative data concerning large floods in Britain is only neutralized by the conservative methods recommended in the Report of the Institution of Civil Engineers Committee (1933) on ' Floods in relation to reservoir practice.'

Problems of water-supply services are often problems of extremes, of predicting the largest and smallest quantities of rainfall and run-off. Can the meteorologist help in the statistical study of these extremes ?

The need to plan water supplies many years ahead of requirement involves forecasting. Are long-term forecasts of rainfall possible, in particular are average annual rainfalls likely to increase or decrease ? Will the proportions of summer and winter rainfalls vary, and similarly will there by any long-term trends in temperature ?

To be of any value, the long-term forecasts must be reliable in the sense that the chance of the occurrence of a more extreme value than that adopted for design must be quite small.

The engineer operating a water-supply undertaking is keenly interested in short-term weather forecasts of cold and dry spells. He particularly would wish to know of the onset and probable duration of such spells and is particularly concerned with a forecast of the probable end of a cold spell. It is at that time that the plumbing systems break down and the water-supply engineer is faced with a large increase in use and waste of water which his works may have great difficulty in satisfying.

The personal approach of the televised weather forecast can be of the greatest importance in warning householders of frost periods and so save them inconvenience and the expense of plumbing repairs, at the same time reducing the peak demand on the water-supply system.

8. Conclusions

The paper has attempted to expose gaps in our knowledge of the hydrological data on which we should depend for the maintenance and extension of a reliable water-supply system.

The main defects would seem to be :

(a) Apathy in regard to river-flow gauging and publication of records.

(b) Insufficient well records and no publication to date of the large amount of data collected compulsorily in recent years.

(c) Inadequate facilities for, and expenditure on, research into the various stages of the hydrological cycle, such as evaporation and percolation.

(d) Very little information available on the effects of temperature, wind, sunshine and vegetation on the hydrological cycle.

By far the most serious lack is in respect of stream gaugings and publication of records – until this defect is remedied, studies of the other branches of hydrology are incapable of being tested, and cannot be fully relied on for practical purposes.

You may consider whether publication of hydrological data should be co-ordinated and whether the co-ordination should be in the hands of a Government agency or a body such as the British National Sub-Committee on Hydrology.

Statistical methods would seem to have their uses in sorting and relating data. What value can be attached to conclusions reached statistically ? In particular what length of time must a record cover to be suitable for use and should conclusions arrived at in this way always be accompanied by a statement of probable error ?

At what value does the coefficient of correlation become an indication that a relationship is of value for prediction as distinct from merely indicating the existence of a relationship ? For practical purposes, it may have to be a value closely approaching unity.

Acknowledgments

I would like to thank the Royal Meteorological Society for allowing me to speak in this discussion. I am grateful for the assistance I have received in providing the illustrations and diagrams from Mr. C. A. Risbridger, Chief Engineer to Birmingham Corporation Water Department and Mr. F. Law, Engineer to the Fylde Water Board. My thanks are due to Mr. C. F. Lapworth and the Institution of Water Engineers for permission to use diagrams from their publications.

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Components in the water balance of a catchment area

By H. L. PENMAN

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1. Introduction

The purpose of this note is to show the amplitudes and phases of the annual changes in monthly rainfall, run-off, storage, evaporation and soil-moisture deficit over a period of five years for the Thames Basin above Teddington Weir. Some of the material has already been published in a survey of changes of storage over the period 1930-1949 (Penman 1951).

2. Basic hydrology

As a preliminary it is convenient, and realistic, to divide the water content of the earth nto two parts. First, ground water or storage which is drawn on by lifting or pumping from vells and boreholes; second, soil water which is removed by evaporation, either from the surface f bare soil, or from root depth by the action of transpiring plants. The importance of the ivision lies in this : except when the water-table (well level) is within a few feet of the ground urface, ground-water level is not affected by surface evaporation. From this division of water ype emerges a corresponding division of catchment behaviour.

Near streams and watercourses – in the ideal river basin – there will be low-lying land in vhich the water-table is high and ground water can contribute to the transpiration demands f growing vegetation. In such areas it is known that the level of ground water, and that of the tream too, will show a small daily oscillation corresponding to the daily cycle of transpiration.

Away from the streams, ignoring a small transition region, the hydrology is very different. tarting in early summer the annual cycle is broadly as follows. The level of the water table vill change in accordance with whatever hydrostatic gradient has been set up in the water-earing rock. The soil-water content will normally decrease throughout the summer as evapora-ion exceeds rainfall, building up a ' soil-moisture deficit ' which in south-east England usually oes on increasing until the end of August or September. This build-up is determined primarily y prevailing weather which determines the potential evaporation rate, and secondarily by the ibility of the plant roots to forage for the water they need. Deep-rooted plants such as trees vill build up a bigger deficit than shallow-rooted plants such as grass, and in any detailed attempt t a water-balance for a catchment some allowance must be made for the distribution of vegetation ver the catchment. But whatever the size of the deficit there eventually comes a month when ainfall exceeds potential evaporation and then the deficit is decreased – *from the top downward* – nd it is not until all the deficit has been wiped out that there is any appreciable downward ercolation of water to depth. When this does occur the surplus water can move down to he water table, and, with a lag depending on the distance to be travelled and the permeability f the soil or rock, the water table will then rise. Normally this rise will persist through the vinter and spring as long as rainfall exceeds evaporation. Then, in early summer, when evapora-ion again exceeds rainfall the water table will once again cease to be dependent on what is appening over it, except for the delayed rise because of water still draining downward.

3. Water-balance equation

The balance equation can be set out simply :

$$W_1 + R_{12} = r_{12} + E_{12} + W_2$$

vhere W_1, W_2 = water content of soil and rock at beginning and end of period

R_{12} = rainfall of period

r_{12} = run-off of period

nd E_{12} = evaporation of period.

Away from streams where the two kinds of water are important it is convenient to set

$$W = S + M$$

vhere S = stored or ground water

M = soil moisture

nd, hence, remembering that only changes can be measured from arbitrary zeros, the general alance equation becomes :

$$R = r + E + \Delta W$$
$$= r + E + \Delta S + \Delta M.$$

n this equation, R is the measured rainfall; r is the measured run-off and is the source of surface vater supplies; ΔS is the increase in storage and is the source of underground supplies. Measure-nent of ΔS is extremely difficult, and for the present note the movements of a well level will e used to indicate phase and relative amplitude from year to year. The term E represents ctual evaporation, but close to streams, and elsewhere for very deep-rooted crops, it will be

the same as the potential evaporation, calculable from weather data. Where shallow-rooted plants dominate, the actual evaporation will be less than the potential, but a basis for making at least an approximate correction for this exists (Penman 1949). The same treatment also permits estimates of ΔM to be made, but as the only state in the soil water régime that can be identified with certainty is that in which M has its maximum value (technically the soil is then at ' field capacity ') it is convenient to assess changes in M as deficits below this maximum. The upper limit of M is thus taken as zero soil-moisture deficit.

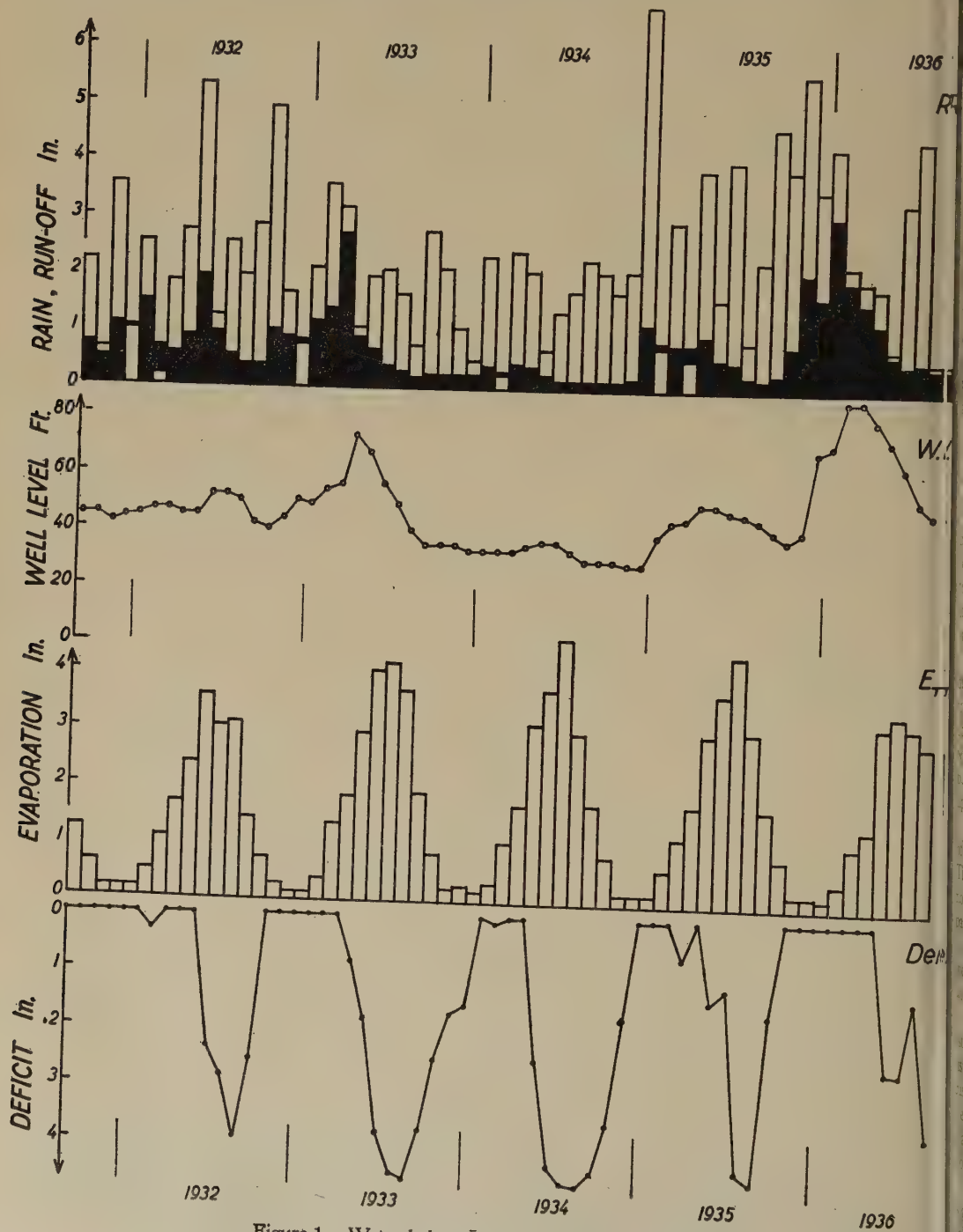

Figure 1. Water-balance components, Thames, 1931-36.

4. Water-balance components

Data for each month from Sept. 1931 to Aug. 1936 are in Fig. 1. This particular period was chosen for two reasons. First, the well level was about the same at the end as it was at the beginning, so that this is a natural period to which the long-term average equation is applicable. Second, it contains an unusually dry winter (1933-4) when well levels showed little sign of winter recovery.

The diagram has observations on the top half, and calculations on the lower. Taking them in order :

R, *r*. Open columns represent measured rainfall over the whole catchment (3,855 mi²). Closed columns represent measured river flow at Teddington, corrected for permitted abstractions. Occasionally monthly run-off exceeds monthly rain, either because of snow-melt or heavy rain late in the previous month.

W.L. This represents the depth of water in a well at Hursley. The mean surface level is about 100 ft below ground level. Hursley is on the edge of the catchment, and the behaviour of this well can only be approximately representative of changes in stored water in the catchment as a whole. Assuming a specific yield of 1/120 (known to be approximately true) then a rise of 20 ft corresponds to an increase in storage of the equivalent of 2 in. of rain.

E_T. The monthly values of potential transpiration are as calculated for the 1951 paper, in which it was suggested that, in effect, these are applicable to about half the catchment. For the other half a correction is needed to allow for shortage of summer rain. The values of E_T as drawn are, therefore, overestimates of evaporation from the whole catchment.

Def. The effect of restricted root range is to reduce the maximum deficit that can be built up. This part of the seasonal trend of deficit is for that part of the catchment in which the restriction is operative. Elsewhere maximum deficits would be bigger.

5. Discussion

Detailed discussion would merely repeat much of what is already in print (Penman 1951), but one internal check on consistency is worth noting. The calculated soil-moisture deficit returned to zero in October 1932, January 1934, December 1934, and October 1935; except for the second of these, there was a clear rise in the well level in each of these months. The exception, in the winter of 1933-34, occurs because the calculated surplus water was very small, and the response of the well was correspondingly slight.

When all the components are taken into the balance sheet, including reduced evaporation on part of the catchment, and bigger deficits elsewhere, it is possible to plot the estimated seasonal trend in storage. This has been done (Fig. 3 of Penman 1951) month by month for 20 years, comparing the estimated storage with the observed levels of three wells, including that of Hursley which is on Fig. 1. The general agreement is sufficiently good to induce a feeling of complacency, but this agreement is merely a result of happy guesses and fortunate assumptions needed to cover up ignorance.

Re-examining Fig. 1, the top section should be free from assumptions. The measurement of rainfall is meteorology, and it can safely be accepted that the gauging and averaging for the Thames catchment provide the most reliable of any data in the balance sheet. River gauging is not part of meteorology, and there is room for improvement in technique, perhaps not at this particular place, but certainly elsewhere.

Measurement of storage is extremely difficult and it may well be that the major help that meteorologists and others could give to the geologist is to get all the other terms in the balance with sufficient accuracy to enable storage to be estimated by difference.

Of these other terms evaporation is as important as rainfall, and is much more difficult to estimate. In the light of experience since 1951 the values of E_T shown in Fig. 1 may be consistently about 5 per cent too high, and have random errors superimposed. To shorten discussion, it can be said that present acceptable errors in monthly values of potential evaporation are greater than the normal midsummer values of river flow. Despite recent advances, natural evaporation remains a major hydrological problem, and this aspect of it is exclusively meteorological.

The other aspect concerns the behaviour of plants and soils, and it is here that ignorance is greatest and there is most need for guesses and assumptions. Though the meteorologist can rightly say that this is outside his field, the primary causes are purely meteorological. The effects concern water supply in two ways. This action of plants in removing water is an asset to the

farmer but is a loss to the water engineer, and, as Fig. 1 shows, the net loss can be equivalent to several months' dry-weather flow of a river. Secondly, the size of the deficit that can be built up is a measure of the possible need for irrigation, and the areas where big deficits occur are those in which the water engineer might at some time in the future be asked to supply extra water at times when other demands are greatest and local supplies are least.

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551.509.67

The modification of rainfall by large-scale cloud seeding

By B. J. MASON
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I have been asked to make a short statement on the present status of rainmaking experiments, to express an opinion on the possibilities of increasing the rainfall artificially over an area of hundreds, or even thousands of square miles. and to indicate what would be involved in the undertaking of such an experiment.

Cloud-seeding experiments are based on three main premises :

(1) That the presence of either ice crystals or comparatively large water droplets is essential to the natural release of precipitation from clouds.

(2) That some clouds precipitate inefficiently, or not at all, because these initiating agents are naturally deficient.

(3) That this deficiency can be remedied by inoculating the clouds artificially with either dry ice or silver iodide to produce ice crystals, or by the introduction of water drops or large hygroscopic nuclei.

There is no doubt that the introduction of these seeding agents into suitable clouds is often followed by observable effects. Such changes are relatively easy to study in the case of individual cumulus clouds seeded from an aircraft because the infected cloud remains a recognisable entity and subsequent changes can be followed both visually and by radar. It has been shown beyond all reasonable doubt, particularly by Bowen's group in Australia, that cumulus clouds in a suitable state of development can be induced to release showers by seeding them with dry ice, when similar unseeded clouds in the vicinity do not precipitate. This type of experiment may well provide important information on the behaviour of cumuliform clouds and on the fundamental mechanisms of precipitation release. It is, however, unlikely to tell us very much about the possibilities of modifying the rainfall from widespread cloud systems extending over large areas.

For experiments on this scale it becomes uneconomic to disperse the seeding agent from aircraft and the general practice is to release silver iodide in the form of smoke from the ground, relying on the air currents to carry it up into the supercooled regions of the cloud mass. Trials of this type have been carried out on a large scale during the past six years, mainly in the United States by commercial operators. The results are the subject of much controversy; unfortunately, judgement has often been clouded by arguments of an intuitive or emotional nature, when critical appraisal of the methods used and a careful analysis of the data would have been more to the point. On the whole, the results at this stage appear inconclusive. From the published material one can find no convincing evidence that very large increases in rainfall can be produced consistently over large areas. Although there is some evidence to suggest that under favourable conditions, marginal effects of 10 to 20 per cent may be produced, in general, the operations have not been continued for a sufficiently long period of time, and have not been designed for use with evaluation techniques sufficiently sensitive to separate effects of this magnitude from the natural variations in rainfall. Certainly, many of the spectacular claims made on the basis of very short-period trials cannot be justified.

The truth is that the assumptions which are made in the operation of large-scale seeding experiments rest on rather insecure foundations, as may be illustrated by our inability to answer the following leading questions.

(1) How often in a given locality, and under what conditions are natural ice nuclei deficient and when, and in what quantities should they be supplied artificially ? Here we have practically no information and it is very difficult to make an intelligent guess.

(2) When silver-iodide smoke is released from the ground does it reach the supercooled parts of the cloud ? If so, in what concentrations, and how will this vary with distance from the source at these levels ? Our knowledge on the turbulent diffusion of particles over distances of tens or hundreds of miles is almost negligible and its study must form an essential part of any serious investigation of the potentialities of large-scale cloud seeding.

(3) For how long does silver iodide remain active as an ice nucleus under atmospheric conditions ? Here also the solution at present is most confused.

But even if we knew the answers to these fundamental questions, the evaluation of these experiments would still present a problem. Because we are still a very long way from being able to compute on a purely physical basis how much rain would fall naturally in a given region, the assessment of the results of seeding would still have to be made on a statistical basis. And, in order to demonstrate convincingly changes of only 10 to 20 per cent which, nevertheless, may be economically significant, the experiment will have to be carried out for a period of years and carefully designed with adequate controls so that the results can be subjected to a rather refined statistical analysis. Perhaps I might indicate very briefly the general lines along which I feel that such an experiment might be planned.

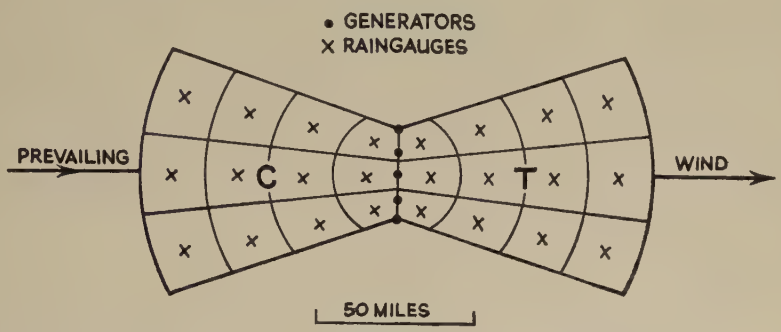


Figure 1. Schematic diagram of target and control areas.

A line of silver-iodide generators spaced about 10 mi apart would be set up roughly perpendicular to the direction of the prevailing wind so that the target area would be as shown diagrammatically in Fig. 1. This area will, of course, be ill-defined at the edges because of variations in wind direction, while the effective range of the silver iodide can only be guessed, but will probably be less than 100 mi on most occasions. As a control area one would, ideally, choose an adjacent region of similar size, shape and topography unlikely to be infected by the silver iodide and one in which historical records show the rainfall to have a high correlation with that of the target area. In Fig. 1, it is shown directly upwind of the generators. One should also aim at providing a roughly uniform and similar distribution of raingauges in the target and control areas.

The method of evaluation generally adopted is to form a regression for the target area rainfall on that in the control area and the assessment of the effects due to seeding takes the form of determining whether there are significant departures from the normal historical relationship. However, this simple regression method has serious limitations. It has been shown that quite different estimates of the seeding effects can be made with different lengths of historical record used in establishing the regressions, while separate regressions can apply for different types of storm. More serious still, if a certain weather trend persists during the seeded period (e.g., a long wet period), highly spurious results may be obtained because the historical averages are not representative of prevailing conditions.

These difficulties may be minimized by carrying out the seeding on a randomized basis. Only half of the suitable storms would be seeded, the selection being made by using, say, a table of random numbers.

A regression can now be formed for the rainfall per storm for the target stations on that for the control stations using the historical records for a large number of past storms of the same general type. This regression is then used to predict the rainfall to be expected naturally in the target area on the occasions of seeding knowing that in the control area. Any difference between the actual and predicted values of the target area rainfall may be attributed to seeding at a significance level evaluated by carrying out the same procedure on the non-seeded occasions (used as a control) when the differences between the predicted and observed rainfall in the target area may be attributed to natural variations. If it should prove impracticable to find a suitable control area, the unseeded occasions may be used as the only control, but the uncertainties in the evaluation will be larger and so the experiment would have to be continued for a correspondingly longer period. Further refinements may be made by sub-dividing the target area and performing a separate analysis for each part and by using more than one parameter in forming the regressions (e.g., more than one control area). Whether these multiple regression techniques will be necessary will depend on the magnitude of the effects to be sought compared with the natural variations, the number of trials and the level of significance judged to be acceptable.

In the few minutes at my disposal I have been able to give only a bare outline of what is involved in the undertaking of a large-scale seeding operation, with little idea of the difficulties involved in trying to meet the requirements of a well-designed experiment. These are manifold. Nevertheless, I am very much in favour of investigating this fascinating and potentially important problem in this country, not solely for the purposes of trying to modify the rainfall, but to try and establish on a proper scientific basis the potentialities of the technique. As a first step, I would urge that a mock experiment be conducted on paper using past data. The idea would be to imagine that certain storms were seeded in the past and to go through the whole operation and evaluation procedures knowing, of course, that no silver iodide had, in fact, been dispersed. This would be an excellent method of getting the feel of the problem, of studying the natural variations in rainfall, of showing up deficiencies in the raingauge network, and would be most helpful in choosing target and control areas and in the design of the experiment proper.

All this will take time; if we have to look for artificial increases in rainfall of order 10 to 20 per cent, the experiment will probably have to be continued for a number of years in order to separate these with high confidence from the natural variations. I should like no one to leave this meeting with the idea that such an experiment is to be undertaken lightly or that a clear-cut result will emerge in a short time, if indeed, at all.

Discussion

Mr. D. Hibbert : Mr. Mason has referred to rain-making experiments in America. I would like to draw attention to the work at present being carried out in Spain. Commercial rain-increasing operations have been going on in central Spain continuously for the past three years and the results suggest that the work is beyond the experimental stage.

The method of checking results suggested by Mr. Mason is probably ideal. Similar methods are, unfortunately, not possible in the Spanish operations. Here the method has been to plot, as a percentage of normal, the monthly rainfall for all stations in Spain and Portugal. Rainfall anomaly charts are then drawn up each month. Plus-anomaly regions can be charted outside the target area – which is comparatively small – but these vary in place from one month to another. For almost all of the past 36 months, however, maximum plus-anomalies can be plotted within the target area. This method is by no means conclusive but the results are significant particularly as they are well supported by comprehensive streamflow data. This work is for hydro-electric undertakings and plans for similar operations in other parts of Europe are well advanced.

The water-supply situation is already serious in many parts of the country, particularly in eastern counties of England. Could not some case be made out for holding trials in these areas where any increased rainfall would be most useful, rather than on Salisbury Plain as proposed ?

Dr. R. S. Scorer : Could one of the water engineers present say how far the water requirements could be diminished by using water more than once ? For instance some sewage works claim that they put out drinkable water into rivers – could not it be returned to the supply ?

Mr. A. J. Whiten : Supplementary to Dr. Scorer's question re the possible multiple uses of water, what is being done in the way of making use of the large quantities of water that are impounded in connection with the hydro-electric schemes now in operation ?

Mr. H. E. Carter : With regard to Dr. Scorer's suggestion on the possibility of re-using water, I recall that some 20 years ago, when stationed in Palestine, I was interested to learn that the extensive Y.M.C.A. centre in Jerusalem had installed plant for collecting, treating and re-using their restricted water supply. The water was used successively in kitchens, bedrooms bathrooms, in the large swimming bath, for cleaning floors, washing cars and finally for watering the flower beds and grass verges which otherwise could not have survived the arid climate of the Middle East.

Dr. E. Wendle-Taylor : In reply to Dr. Scorer I should like to state that water is used more than once in this country. For example, the River Thames is used as a source of domestic water supply; water is abstracted at one point by one town and discharged as effluent below the town to be abstracted again further downstream by another community and finally abstracted by the Metropolitan Water Board for the consumption of the people of London. Another instance is the River Rhine which is used many times over and even by more than one nation.

I would like to take this opportunity to ask Mr. Mason whether there is any toxic hazard in the use of silver iodide to promote artificial rain. Silver salts can be cumulatively toxic to human beings and I am interested in the concentration of silver likely to be precipitated in the rain.

Mr. B. J. Mason (*in reply*) : I regard the ' percentage of normal ' method of evaluation described by Mr. Hibbert as highly unsatisfactory. It may give highly spurious results since natural precipitation for a month, or any relatively short period of time, is generally either above or below the average taken over some longer period ; in other words, the weather is so variable that the historical normal is rarely relevant. It must also be remembered that commercial operators generally seed only when conditions are favourable for precipitation, therefore rainfall above average during a wet month is highly probable with or without seeding activities.

In answer to Dr. Wendle-Taylor, I would estimate the optimum concentration of silver iodide in the supercooled region of the cloud to be about 10 particles or about 10^{-14}g per litre of air. The corresponding concentration in rainwater would be of order 1 part in 10^{17}. Thus the total intake in a long life would not exceed 10^{-12} g.

Mr. E. N. Vellacott : I would like to ask Dr. Buchan whether seepage from one aquifer to another ever reaches significant proportions, and over what distance such a transference of water might occur ?

Mr. J. M. Craddock : I would like information on whether sea water will penetrate far under land through porous strata, and if so, whether it remains salt, or forms a usable and inexhaustible reserve.

Dr. S. Buchan : Extreme values, to which Mr. Halton Thomson has referred, are important to the engineer responsible for impounding surface water, but of less value to the ground-water engineer who can work with average values provided that the aquifer holds enough water in storage to maintain supplies throughout drought years.

Over the last 40 years the amount of ground water abstracted for public supply in England and Wales has been doubled. Errors in estimating rainfall, which were of little consequence at the beginning of the century when the amount of ground water used was far below that of replenishment, may make all the difference between an area being over-developed or under-developed to-day, as the rainfall information is fundamental in planning the development. The British Rainfall Organization produces data more accurate than elsewhere in the world, but now we ask for information to be even more accurate than it has been in the past. Can Dr. Glasspoole hold out any hope that the errors in estimating rainfall can be reduced ?

Mr. Rowntree appears to be satisfied that only a minute percentage of capital expenditure should be spent on water-supply investigations. Surely this is wrong. To spend money on preliminary investigations such as measuring rainfall, percolation and run-off, as well as storage and movement of ground water might result in substantial saving of capital expenditure. His reference to well sinking in scheduled areas is not quite correct. Section 14 of the Water Act of 1945 does not exempt anyone from the need to have a licence before sinking a well to a depth less than 50 ft below the surface. The exemption from requiring a licence is where water is to be used solely for domestic purposes. The delay, mentioned by Mr. Rowntree, in making new geological maps available is due to the need to replace plates and stocks of maps destroyed by enemy action and to new commitments which have reduced the number of staff engaged on field surveys. The inability of the Geological Survey to publish hydrogeological data is due to shortage of staff.

Dr. Penman does not appear to take into account the movement of water through the strata from one part of a basin to another. In a paper on the water balance of the Stour he did not draw attention to the storage water which moved through the aquifer within the area with which he was concerned. Movement of this nature might account for the loss of a considerable volume of water without any apparent change in storage as measured by movement of water level.

Although Mr. Mason indicates that it may not be possible to increase the total amount of rain, an important contribution would be made if it were possible to cause precipitation on the land of rain that otherwise would have fallen in the North Sea.

In reply to Mr. Vellacott I would say that ground-water does move from one catchment basin to another – an instance of this has been described by the Metropolitan Water Board in their 32nd Annual Report on ' Chem. and Bact. Exam. of London Waters ' (1937, pp. 89-93) - and from one aquifer to another through so-called impermeable strata due to differences of pressure. The quantities moving through the impermeable strata may be small per unit area but great in the aggregate (Wisler, C. O. and Brater, E. F., *Hydrology*, New York, 1949, pp. 222-3).

Replying to Mr. Craddock, Roberts (' Experiments on filtration of sea water through Triassic Sandstone,' *Report of the 38th meeting of the British Association* 1879, pp. 397-401) claimed that Bunter Sandstone removed chlorides by filtration but showed that it lost this property as salt water continued to be passed through it.

Sea water rarely penetrates inland because the water level is generally higher under the land but direct percolation of more than one mile through permeable strata is known in coastal districts where the water level has been lowered on the landward side by pumping. The unfiltered water remains salt although it may be diluted by admixture with the fresh ground-water of the land mass.

Mr. L. C. W. Bonacina : I was much interested in Dr. Buchan's reference to the Colne-Lea system because of the remarkable series of swallow holes at Water End, North Mimms, on the Middlesex-Hertfordshire border. The water, engulfed in chalk, overlain by glacial drift, finds its way into the Lea as proved by fluorescein tests, whereas at flood time its overflow channel drains in the opposite direction to the Colne.

It is well known that we are making excessive demands on our moderate rainfall such as would have staggered the Victorians who seemed to manage very creditably without bathrooms. These demands are unfortunately coming more and more into open conflict with the needs of what Mr. Rowntree calls ' amenity ' which includes scenery. This aspect of the matter should not be disregarded inasmuch as man does not live by bread alone. It will be recalled that a few years ago fierce opposition was aroused by the Snowdon hydro-electric scheme which would have destroyed the mountain scenery with all the majestic rhythm of a heavy rainfall. In the vaster regions of the Scottish Highlands there is far more scope for enterprises of that character without detriment to the scenery. As a member of the Dartmoor Preservation Association I do know of the constant anxiety felt over the growing water needs of the S. Devon seaside resorts.

Dr. S. K. Pramanik : The impounding of the whole amount of water and allowing the flow of a small amount to meet regular requirements would prevent large flows which scour the river channel. This may lead to a deterioration and silting-up of the channel, and floods when they occur would be more widespread and serious. Are such effects observed in this country ?

Dr. Glasspoole spoke of estimating the rainfall on mountain ranges from the properties of the air mass and its ascent. Does he really think this is a practical proposition ? We know how very difficult it is to get a rough idea of the rainfall, even over a large area, from air-mass considerations, and for estimating the maximum possible rainfall over an area the method of adjusting the large storms to the highest possible dew point has been employed.

Have the probabilities of rainfall of different amounts over different periods of 1 day, 2 days, 3 days, in river catchments, been worked out, and if so, has this been done by fitting suitable curves to the frequency distributions ? Also, have estimates of the maximum rainfall occurring say once in 500 or 1,000 years been made, and if so how have they been made ? Have estimates of maximum rainfall from air-mass considerations been made ?

Increasing the rainfall, by seeding, over some areas would I presume mean decreasing the rainfall over some other areas, though to what extent it is presumably not possible to say. Increasing rainfall over some areas would, therefore, imply a redistribution of rainfall, at least, to some extent, over those and other areas.

Dr. J. Glasspoole : In reply to Dr. Pramanik, I think we should expect progress in defining the amount of rainfall on the summits of mountains from physical reasoning, taking into consideration measurements made at lower altitudes, wind distribution and orography. In this country estimates of the daily amounts of rainfall are available for only a few catchment areas, including the

Thames Valley above Teddington since 1911. Although frequencies have been evaluated, attention has been directed to defining the areas covered by all heavy rains, irrespective of catchment areas, rather than the frequencies over individual areas. Such summaries give an indication of the worst conditions which might be expected, if such heavy rains should fall within the catchment areas. Frequencies of the maximum daily amounts at individual stations are available, but the interpretation of the data, from formulae available, to give the maximum once in 500 or 1,000 years is uncertain for a number of reasons, including climatic changes. The amounts of rainfall which have occurred during various types of meteorological situations have been defined, e.g., in ' Meteorological conditions in heavy orographical rainfall in the British Isles ' by Douglas and Glasspoole, *Quart. J. R. Met. Soc.*, **73**, p. 11, 1947.

Mr. W. Allard : On the continent of Europe some experimental areas (e.g. one at Alrance in southern France by the French Electricity Authority and one in the Harz Mountains by the German Federal Hydrological Institute, at Koblenz) have been established for the determination of the hydrological water balance and other related studies. For several reasons such areas tend to be small, for example, three to four square miles in extent. I should like to know whether the establishment of one or more such areas in this country is desirable or whether there is thought to be some relevant hydrological phenomenon whose measurement will be impossible. I am under the impression that some work is already being done on an area in Bedfordshire that might serve as a beginning in the matter.

One aspect of water resources, viz., water temperature, may not need to be observed for many hydrological studies. As regards rivers, moreover, it may be argued that the water temperature can be calculated from the observations of air temperatures and other meteorological phenomena. Nevertheless it seems to me that some systematic and direct measurements should be made and the records published, since they can become at any time of importance to one or more of several water interests. Urgency may be attached to this matter as atomic energy is apparently to be made available for industrial purposes in the not distant future and greater demands may then accordingly be made upon the cooling resources of our rivers.

Mr. R. G. Veryard (*communicated*) : I would like to take this opportunity to pay tribute to the many hundreds, in fact thousands, of observers all over the country who voluntarily maintain raingauges, take routine observations and send reports regularly to the Meteorological Office. All of us who are concerned with the application of rainfall data owe them a debt of gratitude; indeed they are performing a valuable national service.

551.515.7 : 551.513 : 551.557

A study of blocking in the Atlantic-European sector of the northern hemisphere

By E. J. SUMNER

(Read 15 December 1954. See *Q.J.*, **80**, p. 402)

MR. R. MURRAY : I found this paper very interesting. However, I think it would have been even more interesting from the viewpoint of practical forecasting if Mr. Sumner had discussed the factors which determine whether a blocking pattern will be progressive or retrogressive. Is Mr. Sumner able to do this now ?

MR. R. G. VERYARD : I think Rex has suggested that ' blocking ' has increased considerably in recent years. Could Mr. Sumner please say whether the distribution of occasions of blocking varied much from year to year over the period he examined ? Can he say also whether there has been a marked absence or excess of blocking this year ?

MR. J. SCHOVE : In some years, such as 1884, 1897, 1901, 1911, 1920, 1933, 1939 and 1940, blocking highs are unusually frequent in N. Europe or the NE. Atlantic. In others, such as 1883, 1903, 1943, 1945 and 1948, blocking is comparatively rare (see chart by H. Brezowsky, H. Flohn and P. Hess in *Tellus*, **3**, p. 193, 1951). Sir Gilbert Walker's studies of the North Atlantic oscillation suggest that this difference in particular seasons extends at least to eastern Canada. Is there any correlation in such years with blocking in other parts of the world (e.g., the eastern Pacific) ? It should be possible to use Sir Gilbert Walker's methods to predict blocking in late Spring when anomalous sea temperatures would be significant factors; Smed's recent tabulations of sea temperatures may be relevant, e.g., in predicting the intensity of the associated north monsoon in Scandinavia (cf. ' Statistics for individual summers,' Schove, *Geogr. Ann.*, Nos. 3-4, 1954). The ' blocking ' maximum between late October and mid-December might depend on the date of effective snow-cover in Scandinavia and any attempt to define snow-cover dates would seem to offer a useful basis for foreshadowing blocking in November. I should be interested to know whether any progress has been made in predicting the intensity of blocking in a given season either in Europe or America. Can any significance be attached to the similarity since *c.* 1880 between curves of 10-yr moving averages of blocking in Europe (*Tellus*, op. cit.) and that of *reversed* sea temperatures south of Greenland ?

MR. H. C. SHELLARD : A point which interested me is that Mr. Sumner has examined charts for a recent 4-yr period and found an average number of days of blocking per annum of rather more than 200, whereas Rex examined charts for an earlier 15-yr period and found an average of rather less than 100 days of Atlantic blocking per annum. Could Mr. Sumner say whether this marked difference arises mainly from the different criteria adopted or whether it indicates a real change in blocking frequency ?

MR. E. N. VELLACOTT : I would like to ask whether there might be a relationship between the sea temperature of the North Atlantic, and the incidence of the ' blocking highs ' mentioned ?

MR. L. C. W. BONACINA : If blocking has any connection with the suspension of strong westerly circulation over the British Isles it is easy to see why May is the month of maximum blocking action, but difficult to understand why the minimum occurs in July definitely rather than August – usually a month of disturbed cyclonic weather from the Atlantic.

DR. R. C. SUTCLIFFE : I welcome Mr. Sumner's paper because he defines blocking as a characteristic pattern of flow and does not beg any questions by selecting cases according to persistence or according to the ' progressive ' or ' retrogressive ' movement of the pattern.

An important conclusion is that this type of flow pattern is a normal rather than an exceptional feature of the northern hemisphere circulation. It is I believe rare in the southern hemisphere and we may conclude that the cause is fundamentally geographical or topographical and not to be expected on a symmetrical earth. Large-amplitude, long-wave distortions of the westerly flow are imposed by the geographical asymmetry and are evidently liable to disrupt owing, no doubt, to a special type of dynamical instability.

PROF. P. A. SHEPPARD : I look for some development of the term ' blocking situation,' or

one of its variants, which will render the term more precise, and more useful, both in descriptive and dynamical meteorology and in forecasting. I appreciate the difficulty in doing this because of the infinity of flow fields, but the embarrassing *richesse* of these fields has not prevented us from classifying usefully into depressions, anticyclones, and so forth, and may not prevent us from identifying, in a rather clear-cut fashion, significant larger-scale features or combinations of features.

I gather Mr. Sumner's definition of a blocking situation is not directly concerned with blocking at all, but is made in terms of integrated zonal motion, albeit qualitative. Could he not pursue this line and tell us, now or later, what *quantitative* description of the field is related to its evolution (or lack of it) ? One senses that it may be something along the lines of meridional amplitude of disturbance though the bifurcation of the upper-flow field on many occasions will evidently complicate matters.

Mr. A. J. Whiten : I notice, in Fig. 4, that nearly all the spells of blocking were initially either progressive or stagnant, and became retrograde during their existence, finally returning to a stagnant or progressive form before breaking down. I suggest that this may be due to the pressure building up on the western edge of the system, owing to the continual supply of new air in the south-westerly flow normal to these latitudes.

The tendency for high pressure to build up with time is well known and progressive movement, along the normal air-flow, in the early stages before the system is well established, is to be expected. As, however, the system becomes more firmly established, the new air being continually fed to the western margin will build up the pressure along this edge more rapidly than the system will move eastwards, so that it will grow rather than move to the west. Later, when the system is decaying, the new air will be insufficient to balance the general decrease of pressure at the western edge, and the system will move away to the east.

It is probably not reasonable to try to account for the movements of all high-pressure systems by the same mechanical trends, so it is not surprising that there should be a few exceptions.

Mr. E. J. Sumner (*in reply, partly communicated*) : In reply to Mr. Murray's question, I have not made a systematic study of the movement of blocking patterns. However, the Rossby wave formula (see Sumner, *Met. Mag.*, **82**, p. 148, 1953) may be applied in cases for which a reasonably sinusoidal upper flow exists round the blocking high or low (which may move quite independently). Irregular or non-sinusoidal patterns cannot be so treated, although a crude estimate of constant-vorticity trajectories may be suggestive, associated with a reasonable extrapolation of the synoptic developments going on at the time.

For Mr. Veryard's information there was remarkably little variation of blocking within the area and period considered. The actual figures (in days of blocking per month) are : 1949, 15·0; 1950 and 1951, 16·5; 1952, 15·5. The corresponding figures for 1953 and 1954 are about 15·0 days. The seasonal distributions are also similar.

Mr. Schove's welcome contribution goes beyond the scope of my investigation. However, Rex, whose study was hemispheric, makes no mention of any relationship between blocking in the Atlantic sector and in the Pacific, where another maximum of blocking is to be found. Pacific blocking shows an almost identical annual distribution to the Atlantic, but the latter is almost twice as frequent. There must be many months (and probably years) when blocking is frequent on the Atlantic and rare elsewhere. I know of no way of predicting the ' intensity ' of blocking at any time, indeed no satisfactory index of blocking intensity has yet been devised. The similarity of blocking in Europe and the ' reversed ' sea temperature south of Greenland is understandable, since blocking anticyclones are most frequent just west of the Greenwich meridian, in which cases the (usually deep) lows which become slow-moving upwind would for long periods be situated so as to channel a cold northerly air stream (and, in spring, masses of ice) down the Davis Strait (Fig. 1 of my paper shows a good example of such a situation).

It is tempting to attribute the seasonal distribution and general behaviour of blocking (my Fig. 6) to a ' preference ' of blocking anticyclones, *once they have formed*, for a relatively cold under-surface, as do both Mr. Vellacott and Mr. Schove. However, the occurrence of blocking highs over the sea in winter (e.g. Feb. 1952, and the recent excellent example of blocking retrogression *from* Scandinavia to the Atlantic in early January 1955) and over Asia in summer, although infrequent, points to other controls.

In reply to Mr. Shellard, Rex's criterion of blocking, which insisted on a minimum (10 days') persistence and size, certainly excluded very many cases which I would have included. However, there has undoubtedly been some change in blocking frequency over the years, with possibly a maximum around 1950-51, but nothing comparable with the 2 : 1 increase suggested by Mr. Shellard's figures.

Mr. Bonacina will see from Figs. 4 and 6 of my paper that much of the August blocking occurred well to the east of the British Isles. The régime upwind of a region of blocking is usually a disturbed westerly or cyclonic type, in keeping with his remarks.

The course of the differences between northern and southern hemisphere circulation patterns is fundamentally topographical and geographical (in the broadest sense) as Dr. Sutcliffe remarks. The most evident and most immediate manifestation of this lies in the much greater strength of the westerlies in the south. Presumably wave-like distortions and instabilities do occur there but, as in the northern-hemisphere winter, owing to the strong zonal flow (as much as the absence of obstacles) fail to stabilize into slow-moving large-amplitude patterns.

In connection with Prof. Sheppard's comments, I have little to add to Section 3 of my paper. If a more precise delineation is required for any specific purpose, I have no doubt that some measurement or combination of measurements (such as amplitude, wavelength, rate of diminution of zonal flow with longitude, etc.) could be devised.

Mr. Whiten is not quite accurate in his opening remarks. Only 21 out of the 53 cases were retrogressive at some time with progression or stagnation at the beginning and end. His ideas are reminiscent of the self-development and self-slowing principles to be found elsewhere in the literature.

551.543 : 551.553.532.59

Short-period variations in surface pressure and wind

By I. J. W. POTHECARY

(Read 15 December 1954. See *Q.J.*, **80**, p. 395)

Mr. J. S. Sawyer : Such periodic variations of wind and pressure as were described by Mr. Pothecary are relatively rare events. I should like to ask whether the conditions in the air mass which transmitted the disturbance were unusual, or whether the source of the disturbance itself was exceptional. If the source covered only a small area, it must have released considerable energy; if it covered a larger area then it is difficult to explain how the linear wave front was formed.

Mr. C. K. M. Douglas : The initiating disturbance described in the paper was on an unusual scale. Extensive and severe outbreaks of thunderstorms associated with really hot continental air do not occur every summer over the English Channel or Brittany. Marked cooling by precipitation at night requires a high cloud base and a dry layer above the surface inversion, which exists over both land and sea in these conditions. Land-breeze convergence might have added to the severity of the storms in the Channel (the nocturnal maximum of rainfall in the Channel due to this cause is recognized by forecasters). Banded structure is a common feature of rain distribution and of clouds up to a high level, but the precise relation between this feature and the type of wave described in this paper is not known.

Mr. J. R. Bibby : Has Mr. Pothecary been able to obtain any data from stations in northern France ? If it were possible to extend the isochrones over that area it would help to determine the position and, perhaps, the rough shape of the originating disturbance.

Mr. E. N. Vellacott : I suggest that the example of a short-period variation in surface pressure and wind quoted, is simply a more extensive form of a fairly common ' wave ' formation, liable to occur whenever a cold front becomes slow-moving or semi-defunct.

Prof. P. A. Sheppard : Has Mr. Pothecary looked into the instrumental evidence on this, or other similar occasions, to determine the distance over which these wave disturbances may be propagated and remain indentifiable ? If the distance is large, several hundred miles, before damping becomes considerable, then the apparent rarity of the phenomenon seems strange, since an air mass capable of vibrating in the observed manner is not unusual, nor is it unusual for a contiguous air mass to possess considerable instability.

Dr. R. S. Scorer : The air is, more often than not, suitable for the transmission of gravity waves and so I do not think that the importance of the inversion lies here, but rather in the fact that the existence of a sharp dividing surface between two air masses made it possible for momentum to be quickly transferred from one mass to the other. In this case the mechanism seems

have been the penetration into the lower layer of down draughts generated by thunderstorms the upper layer, causing a blocking of the motion of the lower layer and the setting up of a avity wave in consequence.

The mechanism for the setting-off of the thunderstorms themselves, over the Channel at ght, is of considerable interest and I have elaborated on the remarks of Mr. C. G. F. Bewsey Some features of the diurnal variation of convection,' *Weather*, **9**, p. 203, 1954) in a forthcoming per, ' The growth of cumulus over mountains *Arch. Met. Geophys. Biokl.* (in press).

With regard to Professor Sheppard's question on how far gravity waves of this kind can avel I would say that since they travel with a speed comparable with that of the wind they ould be expected to travel distances comparable with the distance downwind of an obstacle which lee waves are found. I have seen these on the Lancashire coast, in a north-westerly ind, set up by the mountains of Ireland, and so the distances which Mr. Pothecary supposes em to have travelled seem very reasonable. I do not think any comparison can be made with essure-jump lines observed in the United States because they are maintained in existence by ecipitation as they progress, and the mechanism is rather different.

Mr. G. A. Bull : I should like to ask whether Mr. Pothecary has any observations of rtical currents from radiosonde ascents made during the occurrence of the waves for comparison ith the phases of the pressure oscillations.

Mr. I. J. W. Pothecary (*in reply*) : The questions of Mr. Sawyer and Prof. Sheppard have eady been adequately answered in the contributions from Mr. Douglas and Dr. Scorer spectively.

I would inform Mr. Bibby that no data were obtained from stations in northern France, which ere all to the south of the front on which the storms broke out. Although a change of wind ith height and an inversion were features of the air mass over northern France neither effect as as well developed as over southern England. Any wave motion induced in the lower levels ould have been dispersed fairly quickly.

Answering Mr. Vellacott I would say that gravity waves similar to those described are known occur when cold fronts accelerate or decelerate and this occasion might be regarded as a special se of this phenomenon. The blocking effect of the outflow of air from the area of intense nunderstorms over the western English Channel could be assumed similar in effect to the celeration of a cold front.

The air ahead of a cold front must contain similar discontinuities to the air over southern ngland on this occasion for the effect to be well developed.

In reply to Mr. Bull I would like to say that the only radiosonde ascent which would have iven useful information about vertical currents was that from Larkhill, but the last of the oscilla- ons had passed shortly before the start of the ascent.

551.577.34 (94) : 551.557.3

Secular changes in the rainfall regime of SE. Australia

By E. B. KRAUS

(Presented by Prof. G. Manley 15 December 1954. See *Q.J.*, **80**, p. 591)

Dr. S. K. Pramanik : No details have been given about the records of rainfall. Did the ites, the exposures, and the types of raingauge remain the same throughout the periods, and, f any change occurred, were suitable corrections applied to the observed data ? If there have been such changes they might in themselves explain the apparent variation in rainfall unless uitable corrections have been applied. For example the summer rainfall curves of Figs. 2 and 3 ould be explained by a deterioration and then an improvement of exposure or by a change to site where the rainfall is different; the curves also show an increase for thirty years and then sort of flattening, which is to be expected as thirty-year running averages were used.

We found when examining the climatic changes in India, that the period of thirty years was rather too long and ten-year running averages were more suitable, particularly when the

period of data was not long. It would be interesting to see whether ten-year running averages als indicate an increase from about 1893 and also for how long the increase continued – thirty yea or ten years.

For study of climatic changes we found a statistical approach quite suitable, by fittin orthogonal polynomials (up to the fifth degree) to the series of data, supplemented by ten-yea running average curves. A brief examination of the normality of the series and of periodicitie preceded the fitting of the polynomials.

Can we say from the data given in the paper that the climatic changes are discontinuou and have occurred about a particular year (1893)? We know that, sometimes, years with rainfa above average are followed by some years of low rainfall, which are again followed by 'above average' rainfall – a sort of fluctuation, which may or may not occur in later years. Tables and 2 can be explained on this basis. Again, if the change took place about 1893, one woul expect this to be reflected from about 1878 – 15 years earlier – in the thirty-year average curve instead of about 1893.

Do the zonal winds at 700 mb and 500 mb give good correlation in seasons other than winter

Dr. R. S. Scorer: Since the period over which annual figures are meaned depends to som extent on the whim of the author, may I put forward a criterion for choosing the period ove which to take mean values? Before stating it, however, it should be remarked that the idea ca be expressed in a number of different ways, the most convenient way often depending on th nature of the data being processed. If the criterion does not indicate a suitable averaging perio then significant results are not to be obtained by meaning.

The criterion is as follows: if the spectrum of fluctuations be drawn with amplitude versu period a 'gap' in the spectrum may be defined as a band of periodicities over which the amplitud is relatively low. If means are taken over a period in the middle of this band, the fluctuations c shorter period are satisfactorily meaned because several of them are included in the period, an the longer-period fluctuations still remain evident. If averaging is carried out over a perio where the amplitude is large in the spectrum, i.e., not in a gap, then a small change in the perio or phase chosen will effect large changes in the curve of means that results. These changes ar less evident if running means are plotted at small intervals of the averaging period but this doe not affect the criterion as a method of choosing the best period over which to take means.

An outstanding exception is provided by an isolated peak in the spectrum such as is due t the annual fluctuations. In this case the astronomical cause of the fluctuations is known and th phase of the averaging can be controlled in relation to the fluctuations.

Mr. J. M. Craddock: Like Dr. Scorer I have been wondering whether 30-year runnin means are the best way of smoothing a meteorological time series, e.g. of annual mean tempera tures, and have been trying another method. Let $f_1, f_2 \ldots . f_n \ldots . f_N$ be the series to b smoothed, and let

$$a_n = (1 - r) \sum_{j=0}^{\infty} r\, f_{n-j} \quad . \quad . \quad . \quad . \quad . \quad (1$$

so that a_n is a geometrically weighted mean of the terms up to f_n. If we use a_n to predict the nex term, the error e_n is given by

$$e_n = f_{n+1} - a_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (2$$

It is easy to see that

$$a_{n+1} = a_n + (1 - r)\, e_n \quad . \quad . \quad . \quad . \quad . \quad (3$$

and by using equations (2) and (3) in turn we find e_{n+1}, a_{n+2}, etc., up to the end of the series. The mean square error in our predictions is $\frac{1}{N-n} \sum_{j=n}^{N-1} e_j^2$ and this is a function of r, the weighting coefficient used in defining a_n. Repeating the calculation with increasing values of r in the range say 0·5 to 0·99 we find the value of r which on average enables the best prediction to be made of one term using a mean of its predecessors. The calculation is possible but tedious by hand, but is well suited for electronic computation. So far the only series investigated by hand calculation has been the Greenwich yearly mean temperatures for which the optimum value of r is about 0·86, a surprisingly low value.

A similar process can be used when fitting a straight or curved regression line, or a sine curve, to an observed time series with a view to predicting future terms of the series.

Mr. D. J. Schove (*communicated*): This paper is particularly interesting to me, as it answers to some extent an appeal I made in 1949 (*Q.J.*, **76**, p. 164) for southern-hemisphere data expressed as moving 30-year means.

At that time I showed that a northern-hemisphere climatic fluctuation could be interpreted as the southward movement of a positive pressure anomaly which affected northern Europe about 1895. This was preceded by decreased, and followed by increased, south-westerlies, with all that that implies in relation to rainfall. As far as the southern hemisphere was concerned the available barometric evidence was sufficient merely to note (my Fig. 4, p. 154) that a similar ' high-pressure ' pulse affected Australia about the 1890's. It is now evident from this paper that in Australia as in England the westerlies were stronger after 1895 than before. Thus it would seem that these pressure-pulses moved equatorwards in both hemispheres (the parallel is not complete, because S. Australia is much nearer the Equator then N. Europe and because the subsequent development of westerlies did not reach its maximum in Europe until the 1920's).

The tendency for pressure trends to proceed from polar to equatorial regions makes it possible to predict rainfall trends. In Britain, trends in N.W. Scotland preceded those in E. England (Cf. my Fig. 6 and my note in *J. Inst. Water Eng.*, **7**, p. 230, 1953). In Australia we should likewise expect the rainfall trends in the extreme south-east, as at Bukalong, to foreshadow those further north-west (as at Deniliquin). This is exactly what has occurred for each season as indicated by Fig. 2 in this paper.

The rainfall records for S.W. Australia (from Perth and Albany since 1877) have been discussed by J. Gentilli (*The Western Australia Naturalist* **3**, No. 7, 1952). The trends for the various months show a systematic rather than a random pattern. Each of the months of the wet season has become wetter, but the dry-season months (i.e., summer or Dec.-Feb.) at Perth have become drier. Is Dr. Kraus able to interpret these facts also as linked with an increase in the upper westerlies after 1895 ? Certainly 1895 is a date when a similar increase in westerliness – which could be regarded as a discontinuous climatic change – began in north Europe.

Dr. E. B. Kraus (*in reply*) : In reply to Dr. Pramanik it is stated that no check on exposures and sites of raingauges has been carried out. However, no systematic change in Australian observational procedure occurred in 1894. Although raingauges may have been moved, it is unlikely that this was in fact done at a large number of stations in that particular year. Practically all the quoted records are from small towns : Deniliquin in particular has less than 6,000 inhabitants; Buckalong is a small hamlet with only a few hundred people. Urban development, a major cause for inconsistent records, is negligible.

Dr. Pramanik, as well as Dr. Scorer and Mr. Craddock, stressed the shortcomings of thirty-year running means as a tool for the study of climatic fluctuations. The writer has the same misgivings and, in a study published in the present issue of the *Journal*, residual mass curves were used instead (p. 198). A standard thirty-year mean remains, however, probably the most useful measure for comparison between different stations.

In agreement with Mr. Schove's comments, it is stated that there seems to be a distinct tendency for the change in the rainfall regime to have occurred at an earlier date in higher latitudes then in lower ones. This applies to both hemispheres and is supported by the graphs accompanying the third paper in this series, ' Secular changes of east coast rainfall regimes ' which is in the course of publication in this *Journal*. The statistical significance of this shift has not been investigated.

Proceedings at meetings of the Society

Ordinary Meeting, 19 January 1955

The following candidates were declared to have been elected that day by the Council :

Fellows

Appleby, Ralph Watson, 10 Camperdown Avenue, Burradon, Dudley, S.O., Northumberland.
Baxter, Sydney S., B.Sc., 9 Bishop Road, Salford 6, Lancs.
Belford, James Oluwola, The Meteorological Office, Lungi Airport, Sierra Leone, West Africa.
Brown, George Henry, 17 Halstead Grove, Catley, Cheshire.
Caton, Philip George Frederick, M.A., Ph.D., 8 Bank Drive West, Shrewsbury, Shropshire (*transfer from Student Membership*).
Franklin, Ernest Reginald, B.Sc., 134 Perne Road, Cambridge.
Graystone, Paul, B.A., 42 Sidney Road, Harrow, Middlesex.
Morris, M/Plt. Peter, 51a Norderstrasse, R.A.F. Sylt, 2nd T.A.F., B.O.A.R. 13.
Nihill, John J., B.A., 9 Hughenden Avenue, Cavehill Road, Belfast.
Rae, Alastair George Anderson, B.Sc., Nonikiln, Alness, Ross-shire, Scotland.
Ramaswamy, Chandrasekhara, M.A., F.A.Sc., Institute of Meteorology, Lindhagensgatan 124V, Stockholm K, Sweden.
Rogers, John Brian, B.Sc., 43 Warwick Road, Heaton Norris, Stockport, Cheshire.
Salmond, Fl./Off., James Steward Readdie, Lindores, Appleby Magna, Burton-on-Trent, Staffs.
Timms, Bernard Alan, 126 Lower Hillmorton Road, Rugby, Warwicks.
Wells, Raymond Mervyn, 20 Wychwood Park, Toronto, Ontario, Canada.

Foreign Members

Hilst, Glenn R., S.B., S.M., 2021 Newcomer Street, Richland, Washington, U.S.A.
La Seur, Noel E., Ph.D., Asst. Prof., Department of Meteorology, Florida State University, Tallahassee, Florida, U.S.A.
Lennahan, Charles M., A.B., M.S., 2552, No. 23 Road, Arlington 7, Va., U.S.A.
McDonald, James E., B.A., M.S., Ph.D., Institute of Atmospheric Physics, University of Arizona, Tucson 25, Arizona, U.S.A.

Student Member

Barry, Roger Graham, 37 Harley Road, Sheffield 11, Yorks.

The meeting took the form of a discussion on 'Airflow over mountains' and was opened by Mr. G. A. Corby, who based his remarks on his paper ' The airflow over mountains; a review of the state of current knowledge' (*Q.J.*, **80**, p. 491, 1954). Mr. A. H. Yates then presented on behalf of Dr. J. Förchtgott, Prague, a contribution on ' The measurement of airflow deformation,' after which the meeting was thrown open to general discussion.

Additional Meeting, 26 January 1955

Prof. C.-G. Rossby, Institute of Meteorology, University of Stockholm, Honorary Member and Symons Medallist, gave a lecture on ' Chemical composition of precipitation as a function of atmospheric circulation patterns,' which was followed by general discussion. A vote of thanks to Professor Rossby was proposed by Dr. R. C. Sutcliffe.

Ordinary Meeting, 16 February 1955

The following candidates were declared to have been elected that day by the Council :

Fellows

Allison, James Reginald, ' Little Orchard,' Finchampstead, Wokingham, Berks. (Life Fellow).
Armstrong, Christopher Rodney, B.Sc., F.R.A.S., Guy's Hospital, London, S.E.1.
Arnott, Donald, 40 Campbell Street, Glasgow, N.W., Scotland.
Blair, Ronald H., 23 Garden Village, Muckamore, Co. Antrim, N. Ireland (*reinstated*; previously elected 1952, resigned 1954).
Hebborn, Capt. Anthony William, 16 Crowsport, Hamble, Hants.
Oliver, John, B.A., 441 Gower Road, Killay, Swansea, Glamorgan.
Squire, Prof. Herbert Brian, M.A., Dept. of Aeronautics, Imperial College, Exhibition Road, London, S.W.7.
Thorne, Ralph, Meteorological Office, Headstone Drive, Harrow, Middlesex.

Foreign Members

Alaka, Mikhail, A., S.M., Ph.D., World Meteorological Organization, 1 Avenue de la Paix, Geneva, Switzerland.
Dryden, Warren A., M.S., 442 43rd Street, Brooklyn 32, N.Y., U.S.A.
Kaciak, William, 69 W 225 Street, New York 63, N.Y., U.S.A.
Rosenbloom, Abe, M.S., 7905 14th Avenue, Hyattsville, Maryland, U.S.A.
Showalter, Albert K., B.A., 2927 Tilden Street, N.W., Washington, D.C.

Associates

Campbell, Lawrence Trelford, Office of Director of Air Services, Department of Transport, Ottawa, Ontario, Canada.
Denison, Paul J., 910 46th Avenue, Lachine, Quebec, Canada.
Garrison, Philip Norman, 313 Avenue Road, Toronto 7, Ontario, Canada.
Hardy, Kenneth Reginald, Meteorological Observing Station, Resolute, Cornwallis Island, N.W.T., c/o Eastern Arctic Patrol, R.M.S., Ottawa, Ontario, Canada.
Mezei, Leslie, 817 Bathurst Street, Toronto, Ontario, Canada.
Thorsteinson, Carl Jacob, c/o 4-455 Agnes Street, Winnipeg, Manitoba, Canada.

Student Members

Nielson, Adrian William, 33 Spen Road, Westpark, Leeds 16, Yorks.
Pelly, Charles Patrick, The Elms, Chickerell, Weymouth, Dorset.

Dr. E. B. Kraus, until recently Meteorological Officer, Snowy Mountains Hydro-electric Authority, Cooma, N.S.W., Australia, gave a talk on ' Meteorology in a large water-engineering project ' and also showed a film on the Snowy Mountains Project. After his talk Dr. Kraus answered numerous questions from an audience of over a hundred, which included many civil and hydro-electric engineers.

Additional Meeting, 25 February 1955

A joint discussion meeting with the Royal Astronomical Society was held in their rooms, the topic being " Hydrodynamical processes in meteorology and geophysics.' Some 80 Members of both Societies were present at the meeting, which consisted of two sessions. The President, Sir Graham Sutton, C.B.E., F.R.S., was in the chair, and the afternoon session was opened by Dr. R. Hide, who spoke on ' Some experiments in thermal convection in a rotating liquid ' and was followed by Mr. J. S. Sawyer on ' The basis of numerical methods in dynamical meteorology '; the meeting was then thrown open to general discussion. At the evening session Mr. H. Charnock showed a film of the rotating parabolic bowl with which Dr. von Arx has carried out experiments into the wind-driven circulation of the oceans. Dr. E. T. Eady then spoke on ' Mathematical aspects of " turbulent " hydro-dynamic processes.' A general discussion followed. A full report of the meeting will be published in *The Observatory*.

Ordinary Meeting, 16 March 1955

The following candidates were declared to have been elected that day by the Council :

Fellows

Douglas, Donald Aubrey, B.Sc., 114 Lichfield Court, Richmond, Surrey.
Gamble, David Henry Alan, ' Fairfield,' Harewood Lane, Romanby, Northallerton, Yorks.
Horrocks, Norman Kendall, B.Sc., 168 Marsland Road, Sale, Manchester.
Jones, Charles Paris, ' Menewethan,' Treverbyn Road, Padstow, Cornwall.
Nelis, Instr. Lt. Richard, B.Sc., R.N., 44 Grangeville Gardens, Finaghy, Belfast (*transfer from Student Membership*).
Pollock, John McPherson, ' Gylen,' Lambert Drive, Dunfermline, Fife.
Shaw, Derek, B.Sc., D.I.C., c/o Research Laboratory, B.T.H., Rugby, Warwicks.
Subramanian, Instr.-Lt. S., B.A., I.N.S. Garuda, Cochin 4, India.

Foreign Members

Bent, Arthur E., Director, Mt. Washington Observatory, Ripley Lane, Weston, Mass., U.S.A.
Megenine, Arkadi, Technical Division, Secretariat, World Meteorological Organization 1 Avenue de la Paix, Geneva, Switzerland.
Simpson, Robert H., B.Sc., M.Sc., U.S. Weather Bureau, Washington 25, D.C., U.S.A.

Student Members

Duck, Frank David, Olde Village Farm, Snape, Bedale, Yorks.
Littler, James Tillyard, 40 Creffield Road, Ealing, London, W.5.

The following papers were read and discussed :

(*a*) ' Two-way relationship between the snow or ice limit and 1,000-500 mb thicknesses in the overlying atmosphere ' by H. H. Lamb (see p. 172 of this issue).

(*b*) ' The concentration, size distribution and production rate of large salt nuclei over the oceans ' by D. J. Moore and B. J. Mason (*Q.J.*, **80**, p. 583, 1954).

Popular Lectures

Four Popular Lectures on ' Clouds ' have been given by Mr. F. H. Ludlam, Imperial College to meet the increasing demand from Education Authorities for the Society's Popular Lectures, primarily intended for sixth-form scholars specializing in Physics, Mathematics or Geography.

The Lectures were held on 27 January, 10, 18 and 22 March; the first two being at the Conference Hall of the London County Council by kind invitation of the Chief Education Officer of the L.C.C. and the third and fourth at the Ealing and Trinity County Schools respectively, by kind invitation of the Education Officer of Middlesex County Council. By arrangement with the Education Officers of Buckinghamshire, Essex, Hertfordshire, Kent and Surrey, scholars from these areas were also present, together with a number of pupils from independent schools.

The total attendance at the four lectures was about 1,200.

Reviews

Methoden und Probleme der Wettervorhersage. By H. Reuter. Vienna (Springer-Verlag), 1954. Pp. viii, 161; 46 Figs.; 3 Tables. 37s. 6d.

Although there are numerous books on meteorology as a science, few books have been written primarily for weather forecasters regarding their technical methods of forecasting. The present book by Dr. Heinz Reuter on methods and problems of weather forecasting is thus especially to be welcomed.

The absence of books on forecasting methods is, at first sight, surprising, but it probably springs from two facts; first, that weather forecasting is primarily in the hands of government services who instruct their own staff in their own training schools and on the job, and secondly from the acknowledged difficulty of expressing the forecaster's experience in written papers. This latter difficulty is becoming less real as more direct application of physical and dynamical ideas is made in meteorology, and as forecasting methods become more objective. It is to be hoped that other books will follow Dr. Reuter's lead.

Dr. Reuter's book is rather more limited in scope than its title might suggest. It is primarily concerned with the problem of constructing prognostic charts for 24 hours ahead. Forecasting the weather itself on the prognostic chart is dismissed in a final chapter of only twelve pages, most of which is given up to the forecasting of temperature. Clearly Dr. Reuter has found that forecasting of cloud, precipitation and visibility is still primarily dependent on the experience of individual forecasters.

The remainder of the book gives a connected and logical account of the methods by which prognostic charts may be prepared and is presumably based on the experience of the German and Austrian meteorological services. An account is given of most recognized theories of synoptic changes, and their results are formulated as rules for use in forecasting. The book is of particular interest in the relative weight given to the various methods and the author's opinion of their value.

The recommended method of constructing prognostic charts differs from British practice by the use of isallobaric maps and the application of ' steering ' to isallobars rather than to the pressure systems themselves, but readers trained in British methods will find little that is unfamiliar.

Dr. Reuter devotes a chapter to recent developments in dynamical meteorology, and their bearing on the forecasting problem. He describes the numerical work of Charney, and the graphical work of Fjörtoft in calculating expected changes on the basis of barotropic flow, along with Sutcliffe's ideas on baroclinic development. This chapter would form a valuable introduction to the subject for a forecaster unfamiliar with the original papers.

J.S.S.

The English Climate. By C. E. P. Brooks. London (English Universities Press), 1954. Pp. x, 214; 27 Figs.; 6 Tables. 12s. 6d.

In this handy volume Dr. Brooks writes with his usual combination of authority and engaging charm on the Englishman's staple topic of conversation. His approach and vocabulary are both remarkably fresh and, in the early chapters particularly, he avoids jargon with a skill which is the greater for being concealed. A clear regional picture emerges, everywhere related to the exigencies and requirements of everyday life. This reader found the treatment of English tornadoes (p. 39) both noteworthy and new but hesitates to record what Lancastrians think of the map of the ' energetics ' of British climate on p. 110. Perhaps it should have been made clearer that such a map can claim validity only for out-of-door activity and one is tempted to conjecture how much of the Londoners' positive index is drained away in the morning scramble to work !

The latter part of the volume makes generally available Brooks's work on weather singularities. It is perhaps inevitable that this leads to emphasis on local catastrophe rather than on the essential kindliness of British climate – gracefully conceded however in a paragraph on p. 200. The British reader, even if near or past his prime, will have to search his memory for perhaps more than two occasions when the dramatic element in British climate was brought home to him personally with any considerable force. The prospective visitor from abroad may well conclude ' no wonder the British can take it ' and cancel his booking forthwith !

On the other hand, even if the reader remains sceptical about singularities on the grounds of the difficulty of defining them both as to type and as to sphere of influence, the fact that he may well be tempted to mark his calendar in advance, with the object of proving the author wrong, will be a stimulus to direct though generalized meteorological observation which can do nothing but good.

P.R.C.

Snow crystals. By U. Nakaya. Harvard University Press, 1954. Pp. xii, 510; 514 Figs.; 188 Plates; 39 Tables. 80s.

In this book Professor Nakaya describes the researches on natural and artifical snow crystals carried out by himself and his colleagues during the last two decades. The material has, for the most part, been taken from papers already published, but the main feature of the book, and presumably the main reason for its publication, is the 1550 beautiful photographs of snow crystals. This collection is undoubtedly the finest in existence; the photographs have not been retouched or specially selected for perfect crystal symmetry, so that they are much more representative of natural crystals than, say, the famous collection of Bentley and Humphreys. Moreover, most of the photographs have been taken by oblique illumination, allowing both surface and internal structure to be seen.

The first part of the book deals with observations on natural crystals, the physical properties of snow and the classification and frequency of occurrence of the various crystal forms. The second part is largely concerned with the author's experiments on artificial snow in which crystals have been grown in the laboratory under controlled conditions of temperature and supersaturation.

Most of the material is purely descriptive – there are no mathematics and few numbers. It is not for the crystal physicist seeking information on the structure and growth mechanism of ice crystals; it is a work of art rather than a scientific treatise. For this reason it should have a wide appeal beyond the confines of meteorology and crystal physics.

It is given to few scientists to amass such a beautiful collection of photographs, much less to have them printed on such a lavish scale. This is a handsome publication and a fitting memorial to Professor Nakaya's lifetime of devoted study in this field. The publishers are to be congratulated on undertaking what must have been a very costly publication, and in keeping the price so reasonable.

B.J.M.

Physique des nuages. By J. Bricard. Paris (Presses Universitaires de France), 1953. Pp. vii, 434; 89 Figs. 1440 Fr.

During the last decade there has been unprecedented activity in the field of cloud physics which is now, perhaps, the most active growing point in meteorology. M. Bricard does not set out to give a comprehensive account of these extensive researches, but rather ' to show as briefly as possible what has been the orientation of these ideas and how they have developed.' This result is not a book for the specialist or the advanced student, but one which, if the treatment had been critical and better balanced, might have served as a useful introduction to the subject.

The first four chapters dealing with the classification of clouds and methods of observation, the general properties of stable and unstable air masses and the formation of cloud systems are sketchy, and somewhat disappointing. One finds the unrealistic and badly drawn diagrams which one expects, but regrets to see, in most popular books on meteorology. There are, however, some excellent cloud photographs.

M. Bricard is happier when dealing with the microphysics in which he reviews briefly the formation of water and ice clouds and the development of precipitation, but he tends to put too much emphasis on theory at the expense of experiment and observation.

A chapter is devoted to the scattering of radiation by clouds and to meteorological optics, and another to the absorption of radiation by clouds. The reviewer feels that the space devoted to these topics is out of proportion to the rather inadequate treatments of the condensation and precipitation mechanisms.

More than one-third of the book is devoted to atmospheric electricity. It is well that this subject should be included because cloud electrification is an integral part of cloud physics. In many respects this section is the most satisfactory part of the book, but again one feels that the space devoted to it is out of proportion and gives the book a lack of balance.

With a rapidly developing subject it is always difficult to decide what may be of permanent value and what is of only passing interest. M. Bricard has quoted from a large number of recent papers, but his choice is rather indiscriminate. If he had been more critical and content to navigate the main stream of events, ignoring the less important tributaries, he would have produced a book of more permanent value, although the present volume hardly deserves its shoddy binding and poor paper.

B.J.M.

Physics of the planet Mars. By Gérard de Vaucouleurs. London (Faber and Faber) 1954. Pp. 365; 61 Figs.; 32 Tables; 9 plates. 50s.

The maximum diameter of the image of Mars at the prime focus of a small telescope may only be 0·3 mm and this is distorted by diffraction effects, optical defects and motions of the earth's atmosphere. Until a few decades ago these hazy pictures of our neighbour planet were studied visually and, since the eye is easily deceived and the images cannot be preserved for examination by others, fantastic speculations were put forward causing many scientists to lose interest in this unusual study. Modern photographic plates, the polarimeter, the spectrometer and the sensitive thermo-couple have changed the status of the subject by giving measurements upon which scientific theories can be based. De Vaucouleurs has carefully collected together all the information which exists and presents it as nothing less than a preliminary survey of Martian meteorology.

It would be easy to become over-enthusiastic about the possibility of studying a second planetary atmosphere, similar to the earth's in many ways, but dissimilar in a few important respects (notably in having a solid rather than water as the principal lower boundary and in containing only very little water vapour); it would also be easy to dismiss the whole thing as over-straining the data and being better left to the Light Programme than a scientific treatise. De Vaucouleurs' attitude is however a commendable mixture of enthusiasm and objectivity. In the main he relies upon measurements for forming his conclusions although occasionally he has to make use of unconfirmed visual observations to prevent his material from becoming too thin. The measurements are of course also open to misinterpretation and the author is often extremely critical about them.

The data are fairly extensive. Ground temperatures can be measured over areas of about one hundredth of the planet's disk although the problems of atmospheric absorption and selective emission call for a more sophisticated treatment. A few wind-speeds have been measured from cloud motions. Three different types of cloud are described together with the violet absorbing layer, the occasional yellow veils, the polar winter mists and the morning and evening mists. Heights have been assigned to some of these phenomena, but to call the schematic arrangement of clouds and layers in Fig. 51 a ' Summary of present knowledge . . . ' is over-optimistic, for the text reveals that the violet layer might be anywhere between the surface and 200 km and two types of cloud can only be placed relative to this layer while the third may not even exist. The ground-level pressure has been measured in several ways and the results are usually consistent with a pressure of about 85 mb. There is some definite evidence on the composition of the atmosphere which suggests that it is mainly nitrogen with some carbon dioxide and possibly a little argon and water vapour.

Some topographical data are also of interest to the atmospheric problem. The polar caps, which may possibly be formed from water frost, seem to distil over from the summer to the winter hemisphere and the ' humidity wave ' is presumably propagated through the atmosphere. The dark patches and streams which are associated with the retreating polar cap and the seasonal changes of colour of the dark areas may also be connected with this ' humidity wave.'

Albedos can be measured rather more accurately than we can measure mean albedos on the earth, and some other fundamental data are available to a more than adequate degree of accuracy, e.g. the incident solar radiation, the value of gravity and the period of rotation.

Finally, part of the book does not deal with the atmosphere but is concerned with the surface composition and the internal structure of the planet.

The task of connecting together these data is essentially the same task as that confronting the terrestrial meteorologist, and here the book is at its weakest. For instance in the chapter on ' theoretical climatology ' much use is made of Milankovitch's theory based upon grey absorption when the limitations of this theory can be exposed by applying it to the earth's atmosphere. However this may be too harsh a criticism, for it is clear that meteorologists have been very slow to take a hand in a study for which their detailed knowledge of the earth's atmosphere makes them particularly suited. A single contribution was made by Sir George Simpson but, this aside, only Dr. Hess of Lowell Observatory appears to have contributed to the subject (and very effectively it would seem from the number of references made to his work).

The author gives many references and tries to do justice to all points of view, and this is one of the most useful aspects of the book, although it sometimes results in a bewildering series of contradictory statements (e.g. the chapter on " The high-level absorbing layer "). References often take the form of very long, literal quotations and those who find this style of writing irritating must be prepared to be annoyed. The plates are disappointing since it turns out that some are photographs of carefully prepared drawings and not prints of astronomical exposures. The text is well-produced and free from obvious errors. The diagrams are clear and well-drawn; the paper is not of good quality but is adequate. The price is reasonable on present-day standards.

This is the first attempt to give a connected and comprehensive account of a subject which promises to be of great importance and interest; it has good and bad points but the defects can be tolerated if only because it is a pioneer effort. Members of the Society with catholic tastes are recommended to read it, to enjoy it and, if they have time, to apply their knowledge to the subject.

R.M.G.

Meteor Astronomy. By A. C. B. Lovell. Oxford (Clarendon Press), 1954. Pp. xiv, 463; 3 Plates; 187 Figs.; 175 Tables. Appendix I, II. 60s.

The book deals with the ' astronomy of meteors '; their physics and related questions, such as heights of appearance, are reserved for another book. It contains a relatively complete description of research in meteor astronomy, although important topics are omitted, such as the problem of reality of radiants.

The research items are described by almost literally following the original sources; not much of an effort toward ' digesting ' the material is made. The method of presentation is therefore lengthy, with a luxurious wealth of detail.

The author is an outstanding authority in radio astronomy, and the reader of Lovell's book will, in the first place, look for results from this branch of science. The highlights are the ' blindfolded ' method of determining radiants, and the diffraction method of measuring velocities, all unhampered by daylight or clouds.

The book covers the visual and photographic techniques of meteor observation, the radio-echo techniques, the astronomical equations of meteor motion, followed by a review of the results for the number, mass, and velocity distributions of sporadic and shower meteors, with cosmological concluding chapters.

One of Lovell's main conclusions is that of the practical absence of interstellar hyperbolic meteors. Unfortunately, no explanation of the method of selection of radar velocities is given. These velocities represent only 5-10 per cent of the total number of observed echoes, and Öpik maintains that the method of selection, although objective, has purged the lists of high velocities preferentially. It is regrettable that Lovell's detailed book does not comment on this essential feature of his own work.

Lovell's suggestion that the residual hyperbolic meteors may be caused by planetary perturbations is unfortunate; the observable fraction of such objects can be easily calculated and is of the order of 10^{-9}; their occurrence is about as probable as that of ghosts.

There are a number of mis-statements in the book, some being caused by careless reference to texts. Thus, on pp. 188, 189, Olivier's criticisms referring to 279 *radiants* of the Arizona Expedition (based on 26,000 individual observations) are represented as referring to a list of 279 *individual velocities* observed by Öpik. This *quid pro quo* is especially disturbing, as it is the only serious attempt at criticizing Öpik's rocking-mirror velocity measurements.

A more comprehensive review of the book has been given in the March issue of the *Irish Astronomical Journal*. In any case, when reading it critically, the research student will find Lovell's book an interesting, stimulating and useful guide to modern meteor astronomy.

E. Öpik

The Meteorology of the Falkland Islands and Dependencies 1944-1950. By J. Pepper. London (Hodgson and Son, published for the Crown Agents, Falkland Islands Dependencies Survey), 1954. Pp. vi, 249; 33 Figs.; XII sketches and diagrams; 15 plates, 119 Tables. 42s.

H. E. the Governor of the Falkland Islands writes in his foreword to this book that it ' contains perhaps the largest single collection of data for the Antarctic and sub-Antarctic, will arouse great interest among students of polar meteorology and . . . be a source of satisfaction to those who took part . . . . .' This is no overstatement, despite the omission of upper air data, reserved for separate publication at a later date. Meteorologists have much cause to be grateful to Sir Miles Clifford for his vision in fostering the F.I.D.S. Meteorological Service, through which Britain uses her Antarctic territories to the benefit of science.

The author writes with the insight of one who would have liked to be there too. A gazetteer account of the observing stations is contributed by G. D. Howkins of the Falklands Meteorological Service, and the collaboration of others with Graham Land outstations experience is acknowledged.

The production is lavish, especially in the copious tables setting forth and collating the data in various ways. The lay-out of some of these tables is downright handsome, a model of clarity achieved through judicious use of space and stressing lines. Nevertheless, the excellent pictures deserved better reproduction and, in spite of a well arranged contents list, the reviewer found the lack of an index something of a handicap.

The discussion (94 pp., including a nine-page summary) is valuable, and its interpretations reflect the newest work. The reader learns, for instance, that Antarctic anticyclones can hardly be permanent but appear rather as repetitive features of the circulation, their positions being evidently subject to considerable variation; that the Antarctic is characterized by an enormous variability, especially in winter, from month to month and in comparisons of the same month in different years (e.g. 25 mb and 20°F differences in mean monthly values). The proportion of easterly wind components barely exceeded 50% at the southernmost station in Graham Land. The lowest surface temperatures in the Graham Land winter do not compare with the N. Canadian and Siberian cold poles, although in summer the difference is reversed – the southern cold pole is more probably in the region of the Australian sector of the plateau. Observed M.S.L. pressures in and near Antarctica have ranged from 927·7 to about 1035 mb, most frequent values being between 975 and 995 mb – perhaps a little higher in the F.I.D.S. sector. Temperatures at the F.I.D.S. bases were between − 39°F and + 47°F; there is no sign of any secular warming in this region.

H.H.L.

Further outlook. By F. H. Ludlam and R. S. Scorer. London (A. Wingate), 1954. Pp. 174; 25 Figs.; 12 Plates. 15s.

It might be supposed that authors planning a scientific book would have in mind some more or less well-defined class of reader, the professional expert, the serious student or some imagined group of the general reading public : a book is intended to be read. But why suppose this ? Why should not the author be like those artists who, we understand, paint only to express and relieve their own feelings, leaving sordid sales to chance ?

No objection can be taken to such an exercise of the freedom of the press but it makes the task of the scientific reviewer less easy; a book with no objective in particular can hardly be judged according to its attainment. Admittedly our present authors say they ' have tried to explain a philosophy of the science of weather study ' but the explanation of a philosophy of a science of a study is not a precise objective and it might even be reassuring to some if we note that there is after all nothing recognizably philosophical about this book unless it is the spirit of enthusiasm and fearless dogmatism which pervades it. Sir David Brunt remarks in his preface that the authors have written the book from a new refreshing viewpoint and this we endorse. While reading the book straight through in two consecutive evenings there were many incidents when the authors, if present, would have been heartily commended or as heartily abused but there was hardly a dull page; this is to a meteorologist a very readable book but is liable too often to stray into topics appropriate only for the technical colloquium to be uniformly interesting to the general reader.

A first chapter on ' The weather we observe ' contains an attractive essay on clouds and, in passing, praise must be given to the really excellent cloud photographs which are interspersed through the book, several due to the authors themselves. Unfortunately the praise cannot be extended to all the line diagrams, some of which are illegible and quite disgrace the publishers.

A second chapter on ' Atmospheric processes ' is slight and comparatively conventional, a little about radiation and a little about circulations; here the polar-front depression is introduced but not ' after Bjerknes.'

In Chapter III with the treatment of turbulence we really meet the authors. It is a difficult subject and is not made to look easy; we felt it was left in a suitable muddle. ' Exploring the atmosphere ' in Chapter IV is mainly about wave motions and soaring flight and the touch of the expert is evident. The case is different in the next two chapters on the ' art ' and the ' science ' of forecasting which cannot get far with never a weather map and which are confined to general pronouncements, sometimes provocative but hardly profound. The unevenness of the work is illustrated by the need to define what is meant by dx and dt in $v = dx/dt$ before trying to discuss numerical computations, the only ' science ' which the authors seem to recognize in forecasting.

The rather long chapter on weather modification makes good introductory reading to an exciting topic which is so new as not to have found a place in many books so far. What cloud seeding may achieve is still a matter for conjecture but it is surprising to find anyone saying at this date that " no one has drawn attention to any parts of them (cloud systems of large-scale atmospheric disturbances) in which seeding could be expected to increase the rainfall." Some of the commercial operators make handsome returns, well-earned or not, by attacking just these disturbances.

There is a last chapter on ' Uncertainties,' an excellent idea in a book on meteorology. Uncertainty principles are highly respectable in fundamental physics but in books on macro-physics, technical or popular, uncertainty, without which life would be intolerable, is usually unmentionable. This chapter is however mainly about forecasting problems and on this subject the authors are not at their best.

In a book of this type where the controversial is sought and the style is forthright (and even slapdash at times) the reader should not be surprised if he quarrels with something on every other page. The reviewer has about fifty points to raise with the authors but has space here to mention only a few.

On p. 20 : ' Three instruments, the barometer and the dry- and wet-bulb thermometers, give him all the information he finds essential.' On p. 21 : ' The anemometers and other instruments are not of much direct use in weather forecasting.' There are of course autographic instruments, raingauges, nephoscopes, cloud searchlights and visibility meters.

On p. 23 : ' up to the present it (the geostrophic wind relation) is the only theoretical result which is constantly put to practical use.' The authors have for the moment forgotten the basic physics, the tephigram, upper air computations, adiabatic changes, as well as all the more or less useful theory of dynamical evolution.

On p. 26 : ' If the sun revolved around a still, ocean-covered earth we may suppose that there would be a simple, steady circulation of the atmosphere.' One may also suppose nothing of the kind.

On p. 29 : ' If these simplifications (small perturbations) are given up then the mathematical treatment of the problem (of large-scale disturbances) becomes so difficult that it is impossible to deal with it.' But current computing methods do deal with large-amplitude disturbances.

On pp. 34-5 : ' In general, anticyclones are reluctant to travel and often their decay sets in when they begin to move.' Yet newly-formed developing anticyclones are in fact usually highly mobile.

We must resist the temptation to continue in this vein with criticisms of deliberately unguarded statements but the reviewer is particularly exercized to explain to himself why these authors, with their known experience of forecasting, should have an attitude to the profession which leads them to point to negative rather than positive attainments. In the first two quotations above we are told that instruments have negligible application and that the whole of its theory lies in the geostrophic equation. These are misleading statements and another on p. 150 makes a similar negative assertion, equally incorrect and equally pointless : ' no great forecaster has ever written a tract that has found a place amongst the teaching texts of meteorology.' Admittedly a full-time general practitioner has little time to write teaching tracts, as indeed university staffs have little chance to gather wisdom about forecasting, but it is wrong to forget that much of the outstanding literature on synoptic meteorology has been provided by men who were forecasters in their time and who might in some cases even have admitted to being great ones : a short alphabetical list must include Bergeron, Bjerknes, Bleeker, Douglas, Namias, Petterssen, Scherhag and Van Mieghem.

This is an uneven and perhaps a hastily written book but enjoyably stimulating and no professional meteorologist should miss the pleasure of reading it. Two afterthoughts : one was sorry to see the sub-tropical jet stream omitted from Fig. 1 and the associated text and mystified by ' a millibar being one millionth of the unit of pressure in the centimetre-gram-second system ' on p. 21.

R.C.S.

Vegetation and watershed management. By E. A. Colman. New York (Ronald Press Company), 1953. Pp. xvi, 412; 6 folding maps; 16 Plates. $7.

This book is one of a series dealing with the water resources of the U.S.A., other volumes having covered ground water, the use of water in industry, the production of fresh water from the sea, and flood control. These studies were developed under the sponsorship of the Conservation Foundation, an independent organization established in 1948 and ' devoted to the conservation of the earth's renewable resources.' Only a first-class work could fulfil the requirements implied by such a statement of origins, and this is what Dr. Colman has in fact written. The book is clearly the product of one individual, free from the aridity of style which characterizes only too many official surveys, and pleasant, even fascinating in many passages, to read. At the same time it covers in full the wide field defined by the title and the introduction, in a manner which is at once authoritative and commendable for its scientific modesty. There is no extravagant claim concerning what is known or what can be done, but a comprehensive and critical account of relevant research material

against a background discussion of the necessary hydrological knowledge, rounded off with a guide to furth research and its applications which will not be seriously out of date for decades. This final chapter is headed ' The opportunities and the challenge,' and it may be remarked that the almost exclusively American origi of the material incorporated in the book, including the overwhelming majority of the 300-odd items liste in the bibliography, does not obscure the fact that the challenge, if not in one important sense the opportunit exists in similar measure in many other countries.

Pending the assembly of equally rich material from elsewhere, the introduction and the main conclusion at any rate are sufficiently general to be of immediate significance throughout the world. ' Water, or th lack of it, sets limits to man's activities.' Starting from this point, amplified by reference to the ever-increasin cost and difficulty of securing adequate supplies for large and expanding industrial communities, and c maintaining adequate control works to safeguard populated and developed flood-plains against disaste the book treats water as a product of the land. The role of vegetation begins at the point where precipitatio strikes the foliage and extends over entire land surfaces to modify the quality, quantity and incidence of wate yield. This is discussed in full to bring out how far management of the vegetation can be used as one means c controlling the yield. Structural control is dealt with only so far as necessary to show how it should be develope as the partner rather than the rival of vegetative control. The discussion of individual items of vegetatio management research forming the body of the book cannot profitably be summarized in a brief review. Thi natural vegetation distribution, croplands, grazing lands and forests all receive attention, and there is a chapte on the deliberate use of fire as a tool in vegetation management. Unless properly controlled, fire used to clea unwanted vegetation can be as disastrous as wildfire in denuding the land, with serious flood and erosio consequences. There are many other examples where good management for water yield would not be achieve by the best management for some other purpose and the author does not fail to indicate, wherever the oppor tunity occurs, that management designed to produce desired effects in one direction may have undesirabl results in another. However, ' grazing, logging and clearing of land for agricultural use are bound to continu in many places where they obviously affect water yield adversely.' More general economic considerationr then enforce a compromise and in these circumstances the most that can be hoped for is that water yield will be ' not unduly impaired.' Whilst the book is fundamentally optimistic and carries the tacit assumptio that all problems are capable of satisfactory solution, an important general conclusion is that ' further researc needed to provide results that have quantitative application to problems of water-yield control, will be time consuming and costly.' Dr. Colman could not have stated and enlarged on this conclusion in such an hones way, had he not been convinced that he has amply demonstrated the necessity for such expensive work He has demonstrated it extremely well in an interesting and valuable book.

A.B.

Plant climate and irrigation. By Sidney A. Searle. Chichester (Chichester Press), 1954. Pp. xi, 155; 32 Figs. 14 Tables. 20s.

Although intended primarily for the scientifically-minded grower, all concerned with what might be termed ' bioclimatology ' will find much of interest and value in this new symposium edited by Sidney A Searle. After a foreword by Dr. J. P. Hudson of Nottingham University, Mr. Searle, contributes Chapter 1 ' Plant climate ' and Chapter 2 ' Environmental factors in the glasshouse,' Chapter 3 ' Growing in a micro- climate ' by Mr. Francis Frampton deals with heating by high-velocity low-pressure hot water systems. Chapter 4 concerns ' The measurement of soil moisture ' and is shared between Mr. Eastwell, who describes electrical methods, and Mr. Searle who discusses the tensiometer; and in Chapter 5 ' The calculation of crop water requirement ' Mr. Searle outlines the methods developed for crops in the open.

Chapter 6 ' The effects of different water régimes on the growth and yield of tomatoes ' by Mr. P. J. Salter occupies some 47 pages and is an account of recent work at Nottingham, whilst in the last Chapter ' Application to commercial growing ' Mr. Searle returns to review the implications of the previous chapters and closes with details of some recent work on osmosis. Three appendices, a comprehensive bibliography of some 200 references and an adequate index complete the volume.

Whenever possible there is a definite attempt to utilize fully existing data on climate and in Chapter 2 some simple relationships are obtained between air and soil temperature. In considering temperature variations over 24 hrs. the author has chosen to work with the range of hourly means rather than the mean daily range, which one would have thought more appropriate (p. 14 and p. 120), although the difference is likely to be relatively small from about mid-March onwards; incidentally (p. 19) some records of radiations are now coming to hand. Chapter 5 appears to be adequate and reasonably up-to-date but the method is not regarded as appropriate to intensive production under glass where insufficient water, for even short periods, can be detrimental to the plant – this danger can, it appears, be largely avoided by making use of tensiometer readings.

Mr. Salter's evidence of a marked reduction in the dry weight of the tomato plant when soil moisture tension is allowed to exceed about 10 cm. of mercury appears contrary to the widespread view that water held between field capacity and permanent wilting percentage is equally available to plants over the whole range. A possible reconciliation of these differences is hinted at both by Salter and Searle. In this and in the last chapter some information is given of the movement of water in the soil and of soil-water/porosity relationships. Mr. Searle also adds an account of recent unpublished work on osmosis by L. Blass which appears to this reviewer to involve some reconsideration of the earlier results. Difficulties of this sort are however

inevitable in a volume such as this in which the reader is to be introduced to the most recent knowledge as well as presented with an appreciation of more familiar results.

In presenting the book Mr. Searle and his collaborators have rendered a very considerable service to all interested in the sciences and practices involved in the protection of food. Throughout, by implication, it is admitted that our growing ability to control the environment in the glasshouse does not make it any less true that, in the long run, Nature 'calls the tune,' and parallel studies of equal thoroughness are also required so that horticultural enterprises may be established in areas where natural conditions are as favourable as possible.

R.W.G.

Africa; a study in tropical development. By L. Dudley Stamp. London (Chapman & Hall), 1953. Pp. vii, 568; 223 Figs.; Tables. 68s.

Professor Dudley Stamp has presented us with a full and lavishly illustrated regional geography of the African continent, well documented and equipped with a useful statistical summary and a generous index. The work is handsomely produced and relatively expensive. Several parts of the text make it clear that it was written mainly for the American reader.

The chapter on 'African climates and the water problem' which is likely to be of most direct interest to readers of this journal contains nothing that is essentially new and though five pages of climatic data are included only mean values for temperature and precipitation are quoted. One is entitled to regret that relative humidity and the mean diurnal range of temperature, so important in the tropics, have not been given more emphasis in this discussion, but Professor Stamp's interests are naturally concentrated on the breath-taking rapidity of modern changes in the African scene.

P.R.C.

Les rayons cosmiques dans leur rapports avec l'électricité atmosphérique, la météorologie et l'astronomie. By A. Dauvillier. Paris (Dunod), 1954. Vol. I, Pp. x, 247; 116 Figs.; 116 Tables. 1750Fr. Vol. II, pp. x; 248-565; 137 Figs.; Tables. 2600Fr.

The author has been engaged in the study of cosmic rays for the last twenty-five years and these two volumes reflect his particular interest in the construction and refinement of apparatus used in their measurement and in the investigation of the geophysical variations of their intensity. The treatment is historical and copious references are provided throughout. The presentation can be criticised for the somewhat excessive amount of space that is devoted to early results and speculations. In places, this tends to obscure the more important conclusions. The work is intended for the non-specialist and largely succeeds in this aim, though a lot of minute technical details in the instrument section of the first volume might with advantage have been omitted. Much of the material has been collected together for the first time, particularly that contained in Vol. II.

The first volume deals with measuring instruments in a very detailed manner; the discussion of ion-chambers seems unduly long, especially since their employment in the study of cosmic rays is now rather limited. There is a surprising remark concerning the magnitude of ionic recombination in cloud-chamber tracks on p. 180. The concluding chapter of this volume deals adequately with the absorption characteristics of cosmic radiation.

The second volume is devoted, with the exception of a chapter reviewing their nature, to the geophysical and cosmological implications of cosmic rays. Geomagnetic variations are discussed in chapter IV and a simplified version is given of the theory of these effects. In chapter VII the author has collected together most of the material available up to the end of 1952 concerning the important study of the time variations of cosmic radiation. This is the section of greatest meteorological interest since an account is given of the extent to which the cosmic-ray intensity can be related to meteorologically significant variables. The final chapter deals briefly with the vital question, as yet unsettled, of the origin of cosmic rays.

K. H. Barker

Waves and Tides. By R. C. H. Russell and Cdr. D. H. Macmillan. London (Hutchinson), 2nd edition, revised, 1954. Pp. 348; Figs., Tables. 25s.

The first edition of this work appeared in 1952 and was reviewed in *Q.J.*, **80**, p. 119, 1954. The present edition corrects a number of minor errors in the first edition.

F.U.

Obituary Notices

Mr. G. BRANSBY-WILLIAMS, M.Inst.C.E., F.R.G.S.

George Bransby-Williams, a Fellow of the Society since 16 December 1931, was born at Killay House, Glamorganshire, in 1872 and educated at Clifton College. After completing his apprenticeship he was engaged in a number of engineering concerns until, towards the close of 1899, he volunteered for active service in the Boer war and proceeded to South Africa as second-in-command of the 1st Volunteer Company of the Welch Regiment. After the capture of Pretoria and the battle of Diamond Hill he was appointed a railway staff officer and placed in charge of the reorganisation and reconstruction of the line from Pretoria to the Portuguese frontier. On demobilisation he was at first occupied on railway work in Wales, but in 1906 was sent by the secretary of state for the colonies to report on the sanitation of Uganda towns. In 1909 he accepted the post of sanitary engineer to the government of Bengal. Retiring from these duties in 1927, he set up as a consulting engineer, initially on his own but subsequently in partnership with Col. F. C. Temple and Mr. G. Bartholomew at Calcutta and Cawnpore. This firm was dissolved in 1944. Mr. Bransby-Williams, who was one of the founders of the Institution of Engineers, India, wrote several technical treatises and contributed frequently to the proceedings of the various professional bodies to which he belonged. A paper by him on ' The rainfall of Assam ' was published in Vol. 58 of this Journal. At one period he was a justice of the peace for Glamorganshire. He died on 17 November 1954.

E.L.H.

Mr. INIGO O. JONES, F.R.A.S.

Few who have held membership in this Society at any time during the 105 years of its history can have devoted so long a stretch of life to constant study of the weather as did Inigo Owen Jones, who died on 15 November 1954 within three weeks of his 82nd birthday. His mother was of the mathematically and philosophically famed Bernoulli family, his father a civil engineer trained in London by (among others) Faraday and Admiral FitzRoy. To this parentage Inigo Jones attributed the intense fascination which meteorology and astronomy had for him from early childhood. Born at Croydon, Surrey, he was taken in his second year to Australia, where his father had been appointed to superintend the construction of a railway. The boy received his education at Brisbane, and while he was still at school his interests and abilities brought him to the notice of Clement Lindley Wragge, at that time colonial meteorologist for Queensland, who recruited him as an observer and taught him what he could. A few years later the father acquired a small farm near the hamlet of Beerwah, some 40 miles north of Brisbane, naming it Crohamhurst after Lord Goschen's Surrey estate. Here Inigo Jones set up the meteorological station which he was destined to maintain for two-thirds of a century, and here, on 2 February 1893, he measured 35·71 in. of rain – the greatest daily fall on record for Australia. Before then Wragge, at the instigation of the postmaster-general for Queensland, had begun to explore the practicability of issuing long-term weather forecasts on the basis of the Brückner cycle and the sunspot period. As a member of Wragge's staff, Inigo Jones was called upon to assist in these studies, and as the years went on he developed such enthusiasm for the work that it became his life's principal concern. Encouragement was not lacking in a land where fore-warning of drought and deluge is of vital importance, and eventually, with financial aid from various interested parties, he established the Queensland Long-range Weather Forecasting Trust, himself directing its activities from his observatory at Crohamhurst.

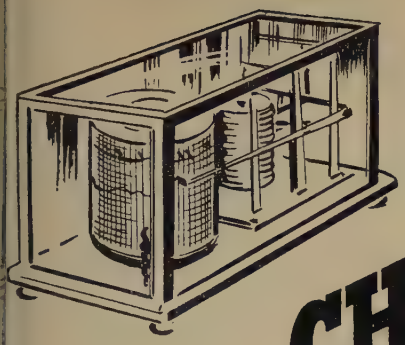
Briefly, Inigo Jones's working hypothesis was that emanations from solar disturbances exert a powerful control on the earth's atmosphere. The sunspot ' cycle,' he maintained, is caused ' electromagnetically ' by the movements of the planets; it is dominated by Jupiter's orbital period of 11·86 years, but irregularities due to Saturn, Uranus and Neptune give the so-called cycle an average length of 11·1 years. Explanations of abnormal weather are thus to be sought not only from the sun but from outer members of the solar system. Particularly important, he found, are conjunctions of the four great planets, which occur at intervals of about 164 years and are often associated with climatic extremes and severe famines. Inigo Jones did not succeed in convincing any substantial body of accredited scientific opinion that the predictionary methods which he evolved from his findings were soundly based; but among less critical sections of the Australian community he had a wide and faithful following. Much was made of his successes, and when, in recent years, a drought or a wet spell arrived a few months earlier or later than he had forecast there was a kindly disposition on the part of farmers and graziers to allow that the quasi-legendary old figure from the hinterland had scored once again. Inigo Jones's other pursuits were stud cattle breeding at Crohamhurst, yachting, painting and violin playing. His Fellowship in this Society dated from 20 November 1935.

E.L.H.

Flt. Lt. W. K. WATSON, D.F.C.

We have to record the death of Flt. Lt. William Kenneth Watson, who was killed in a flying accident off Cuxhaven on the night of 3 June 1954. He was born at Brisbane, Queensland, in 1922 and after serving with the Royal Australian Air Force in England from 1940 to 1945 he joined the R.A.F. in 1949. Photographical reconnaissance work was allocated to him and it was while engaged on these duties in a Canberra aircraft that he met his death. Flt. Lt. Watson became a Fellow of this Society on 16 March 1949.

E.L.H.



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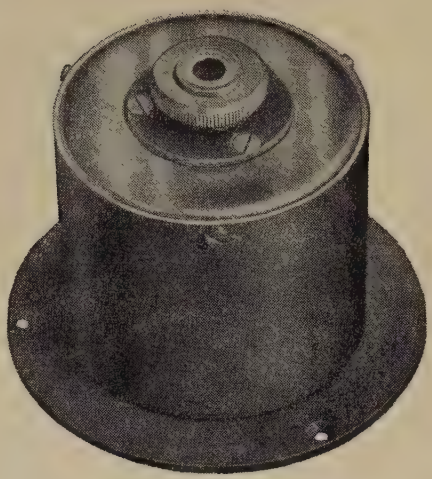

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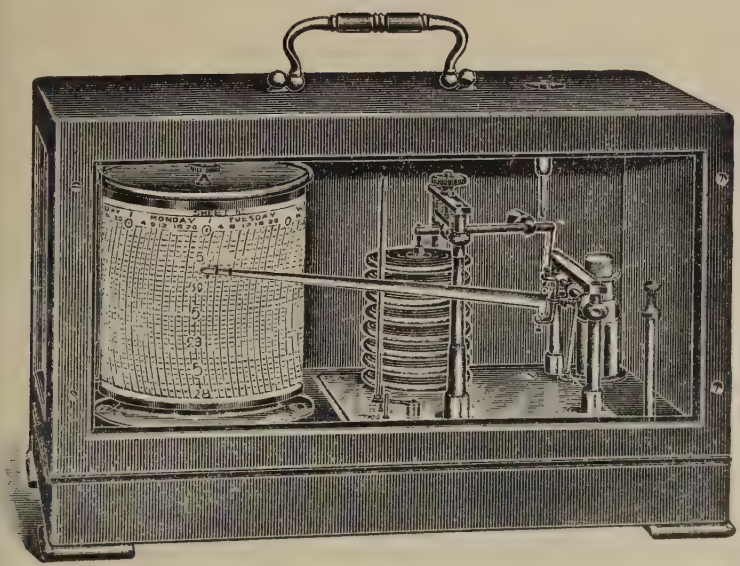

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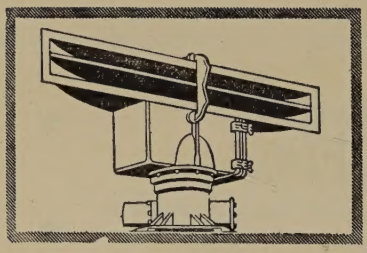

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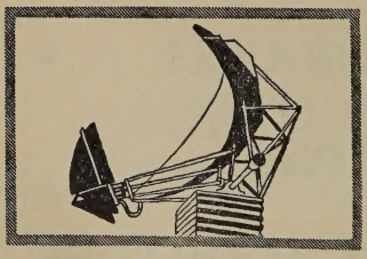

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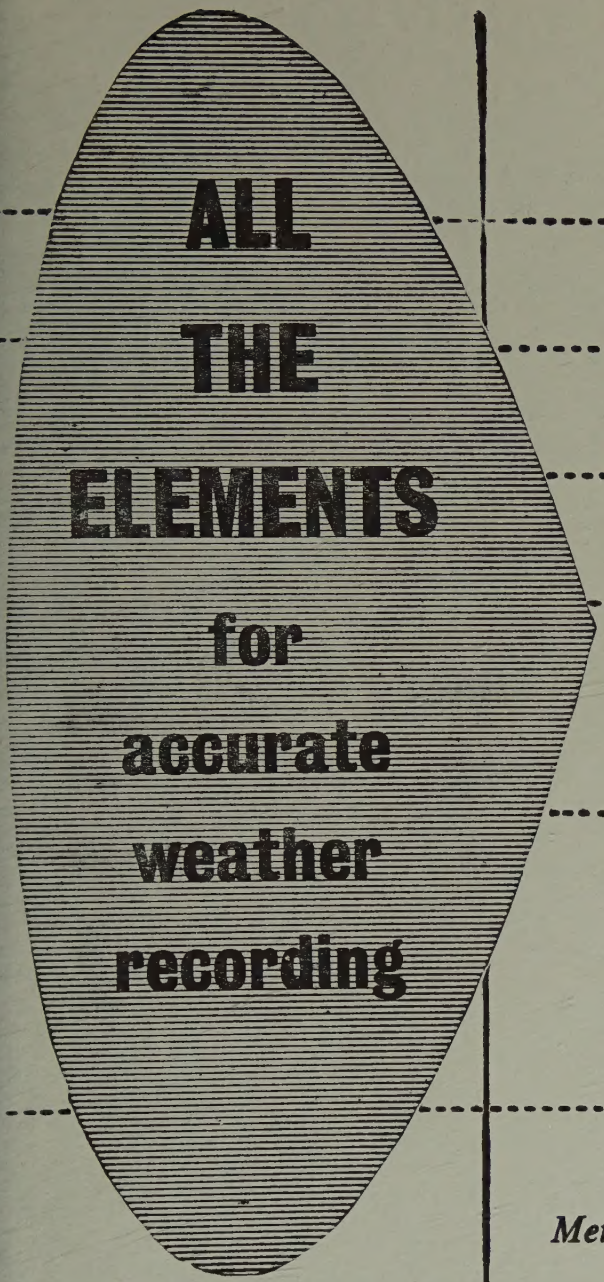

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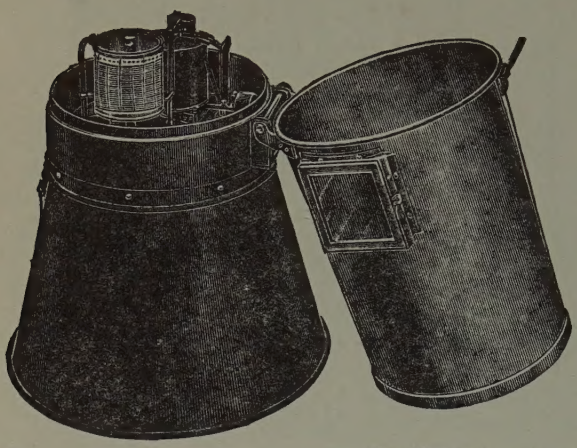

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